a)

S is the maximum value of the N - values.

Thus the most probable value of S is the $1-\frac{1}{N}$ quantile of V.

$$P(V > x) = \frac{1}{N}$$

$$\frac{\lambda^{\alpha}}{(x+\lambda)^{\alpha}} = \frac{1}{N}$$

$$(x+\lambda)^{\alpha} = N\lambda^{\alpha}$$

$$x = N^{\frac{1}{\alpha}}\lambda - \lambda$$

$$= \lambda(N^{\frac{1}{\alpha}}-1)$$

Thus the most probable value of S is $\lambda(N^{\frac{1}{\alpha}}-1)$. It will increase as N at a polynomial order.

b)

Similar to a)

let

$$e^{-x} = \frac{1}{N}$$

$$-x = \log \frac{1}{N} = -\log N$$

$$x = \log N.$$

Thus the most probable value of s is $\log N$.

c) For a) $S_1(N) = \lambda(N^{\frac{1}{\alpha}} - 1)$

For b) $S_2(N) = \log N$

$$\frac{S_1(N)}{S_2(N)} = \lambda \frac{N^{\frac{1}{\alpha}} - 1}{\log N} \to \infty \quad \text{as } N \to \infty.$$

Thus Lomax distribution leads to a faster increase of $S(N)$ as N increases.