(1) The time between failures of our video streaming service follows an exponential distribution with a mean of 20 days. Our servers have been running for 16 days, What is the probability that they will run for at least 46 days? (clarification: run for at least another 30 days given that they have been running 16 days). Report your answer to 3 decimal places.

Solution:

The Probability Density Function (PDF) is:

$$f(x) = \begin{cases} 0.05 \cdot \mathrm{e}^{-0.05 \cdot x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The Cumulative distribution function (CDF) is:

$$\mathrm{F}(x) = 1 - \mathrm{e}^{-0.05 \cdot x}$$

Let event A be that the servers have run for 16 days without failure; Let event B be that run for at least another 30 days without failure given that they have been running 16 days.

Then we have

$$\Pr[B|A] = \frac{\Pr[AB]}{Pr[A]} = \frac{\Pr[B]}{Pr[A]} = \frac{\mathrm{e}^{-0.05 \cdot 46}}{\mathrm{e}^{-0.05 \cdot 16}} = \mathrm{e}^{-1.5} \approx 0.22313$$

(2) A company is going to launch an exclusive high-end smartphone. They rely on a bold marketing strategy that offers a lifetime warranty for replacing the first failure caused by normal use in each smartphone sold.

The engineering team does a precise accelerated aging test on 82 smartphones and detects that 5 fail. The test is precise in simulating the real conditions of a lifetime of use.

A 0.061 probability of failure is just within the long-run budget for the marketing campaign and everyone in the company is in good spirits, until someone in the analytics department warns them that the probability of failure for each smartphone could be much higher. For example, if the true probability of failure was 0.0732 the budget would be greatly exceeded (by 20% !) and the company would have to close. What is the probability of having observed the results of the aging test, or even better results (less failures), if the true probability of failure is 0.0732

Please use the normal approximation and introduce the answer to 3 decimal places.

Solution:

When using normal approximation, the mean of the normal distribution is $82 \times 0.0732 = 6.0024$, the variance is $82 \times 0.0732 \times 0.9268 = 5.56302432$. So the PDF is:

$$f(x) = \frac{1}{\sqrt{2 \cdot \pi \cdot 2.3586}} e^{-\frac{(x-6.0024)^2}{4.7172}}$$

Then we have

Pr[the fail number in the test is Less than or equal to 5] = $\int_0^5 \frac{1}{\sqrt{2 \cdot \pi \cdot 2.3586}} e^{-\frac{(x-6.0024)^2}{4.7172}} dx \approx 0.0045$