To find the differential of e^{X+Y}, we can use Itô 's lemma, which states that for a continuously differentiable function f(x,t), the differential df satisfies the following stochastic differential equation (SDE):

$$df = \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\sigma_x^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}\sigma_y^2 + \frac{\partial^2 f}{\partial x \partial y}\rho\sigma_x\sigma_y \right) dt + \frac{\partial f}{\partial x}\sigma_x dX + \frac{\partial f}{\partial y}\sigma_y dY,$$

where X and Y are 1-dimensional Brownian motions with correlation ρ, σ_x and σ_y are the standard deviations of X and Y, respectively.

Applying Itô 's lemma to $f=e^{X+Y}$, we get:

$$df = \left(\frac{\partial f}{\partial x}\sigma_x + \frac{\partial f}{\partial y}\sigma_y + \frac{1}{2}(\sigma_x^2 + \sigma_y^2 + \rho\sigma_x\sigma_y)f \right) dt + f dW,$$

where W is a 1-dimensional Brownian motion defined by W=X+sqrt(2)Y.

To find the differential of $[W,X]^T$, we can apply Itô 's lemma directly. Since W and X are both 1-dimensional Brownian motions, we have:

$$d[W,X]^T = \begin{bmatrix} dW \\ dX \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} d[X,Y]^T.$$

Using the definition of $[X,Y]^T$ as a 2-dimensional Brownian motion with correlation ρ, we have:

$$d[W,X]^T = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} dX \\ \rho dX + \sqrt{1-\rho^2}dY \end{bmatrix} = \begin{bmatrix} dX + \frac{1}{\sqrt{2}}(\rho dX + \sqrt{1-\rho^2}dY) \\ \frac{1}{\sqrt{2}}dX + \frac{1}{2}(1+\rho)dY \end{bmatrix}.$$

To simplify this expression, we can use the fact that W=X+sqrt(2)Y to eliminate dY:

$$dW = \frac{1}{\sqrt{2}}dX + \sqrt{2}\frac{1}{\sqrt{2}}dY, \quad dY = \frac{1}{\sqrt{2}}dW - \frac{1}{\sqrt{2}}dX.$$

Substituting this expression for dY into the differential of $[W,X]^T$, we get:

$$d[W,X]^T = \begin{bmatrix} dX + \frac{1}{2}(1+\rho)dY \\ \frac{1}{\sqrt{2}}dX + \frac{1}{2}(1-\sqrt{2}\rho)dW \end{bmatrix}.$$

Substituting dY and simplifying, we obtain:

$$d[W,X]^T = \begin{bmatrix} \left(1+\frac{\rho}{\sqrt{2}}\right)dX + \frac{1}{\sqrt{2}}dW \\ \frac{1}{\sqrt{2}}\left(1-\sqrt{2}\rho+\frac{\rho}{\sqrt{2}}\right)dX + \frac{1}{\sqrt{2}}\left(1-\sqrt{2}\rho\right)dW \end{bmatrix}.$$

Therefore, the differential of $[W,X]^T$ is given by:

$$d[W,X]^T = \begin{bmatrix} \left(1+\frac{\rho}{\sqrt{2}}\right)dX + \frac{1}{\sqrt{2}}dW \\ \frac{1}{\sqrt{2}}\left(1-\sqrt{2}\rho+\frac{\rho}{\sqrt{2}}\right)dX + \frac{1}{\sqrt{2}}\left(1-\sqrt{2}\rho\right)dW \end{bmatrix}.$$

To find the differential of e^{X+Y}, we can use the Itô formula, which tells us that for any function $f(X_t,Y_t)$ of a 2-dimensional Brownian motion, its differential is given by:

$$df = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial X}dX + \frac{\partial f}{\partial Y}dY + \frac{1}{2}\left(\frac{\partial^2 f}{\partial X^2} + \rho\frac{\partial^2 f}{\partial X \partial Y} + \rho\frac{\partial^2 f}{\partial Y \partial X} + \frac{\partial^2 f}{\partial Y^2}\right)dt.$$

Evaluating the partial derivatives and simplifying, we obtain:

$$df = e^{X_t+Y_t}\left((dX_t + dY_t) + (\rho+1)dX_t + \frac{1}{2}(2\rho+1)dt\right).$$

Therefore, the differential of e^{X+Y} is given by:

$$de^{X+Y} = e^{X_t+Y_t}\left(\left(\rho + \frac{1}{\sqrt{2}}\right)dX_t + dW_t + \frac{1}{2}(2\rho+1)dt\right).$$

Next, we apply Itô 's formula to find the differential of W. We have:

dW = dX + sqrt(2)dY = dBx - ρBxdx + sqrt(2)(dBx + dBx) - sqrt(2) ρBxdx + sqrt(2) ρ(Bx+By)dt

where we have used the fact that dBx and dBy are independent Brownian motions and have applied Itô 's lemma to each component of X and Y. We also used the fact that dBx^2 = dt and dBy^2 = dt.

Simplifying, we get:

dW = (1+sqrt(2)ρ)dBx - ρBxdx + sqrt(2)ρBydt

Therefore, the differential of $[W,X]^T$ is:

$[dW, dX]^T$ = [(1+sqrt(2) ρ)dBx - ρBxdx + sqrt(2)ρBydt, dBx]

Note that the covariation term is zero since dBx and dBy are independent.

Therefore, the differential of $[W,X]^T$ simplifies to:

$[dW, dX]^T$ = [(1+sqrt(2)ρ)dBx - ρBxdx + sqrt(2)ρBydt, dBx] = [(1+sqrt(2)ρ)dBx, dBx] - ρBxdx + sqrt(2)ρBydt

where we have used the fact that dBx and dBy are independent Brownian motions, and that dBx^2 = dt and dBy^2 = dt.

Hence, the differential of $[W,X]^T$ is:

$[dW, dX]^T$ = (1+sqrt(2)ρ)dt $[1, 1/sqrt(2)]^T$ - ρBxdx $[0, 1]^T$ + sqrt(2)ρBydt $[1, 0]^T$

where we have written the result as a linear combination of dt, dx, and dBx.

This is the desired form of the differential. Note that the first term is a drift term, while the second and third terms are diffusion terms.

In summary, we have found the differential of $[W,X]^T$, where W = X + sqrt(2)Y and $[X,Y]^T$ is a 2-dimensional Brownian motion with correlation coefficient ρ. The differential is:

$[dW, dX]^T$ = (1+sqrt(2)ρ)dt $[1, 1/sqrt(2)]^T$ - ρBxdx $[0, 1]^T$ + sqrt(2)ρBydt $[1, 0]^T$

This result can be used to analyze the behavior of $[W,X]^T$ over time and to compute various statistical properties of this process.