

- (a) The total mass of the system is $m = 1000 + 200 + 80 = 1280$ (kg)

For a damped oscillation with equation of motion being

$$mx''(t) + cx'(t) + kx(t) = 0$$

where m, c, k are mass, damping factor and spring constant respectively, the general solution to it is known to be

$$A \exp\left(-\frac{ct}{2m}\right) \cos(\omega_{\text{damp}}t + \phi)$$

where A, ϕ are amplitude and phase factor depending on the initial conditions, which we don't care about at all, and ω_{damp} is the damped angular velocity given by

$$\omega_{\text{damp}}^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$$

Now we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}} = 1(\text{s})$$

$$\exp\left(-\frac{cT}{2m}\right) = \frac{1}{5}$$

We can solve out

$$c = 4120.16 \text{ (N} \cdot \text{s/m)}$$

$$k = 53847.9 \text{ (N/m)}$$

- (b) Given the velocity v , the bump will change as

$$y(t) = A \sin\left(\frac{\pi vt}{3}\right)$$

It means that if we treat moving on a bump as an external force, the frequency of this external force is $\omega_{\text{ext}} = \pi v/3$.

The maximum amplitude will happen at practical resonance given by

$$\omega_{\text{ext}} = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \frac{\pi v}{3} = 6.283$$

which gives $v = 6$ (m/s)

The maximal amplitude is given by

$$\frac{A}{\sqrt{(k - m\omega_{\text{ext}}^2)^2 + (c\omega_{\text{ext}})^2}} = 0.001497 \text{ (cm)}$$