4. (a) Denote by $g(n)$ the number of gannets in the n-th year. We have

$$g(n+1) = 1.03g(n) - 6$$

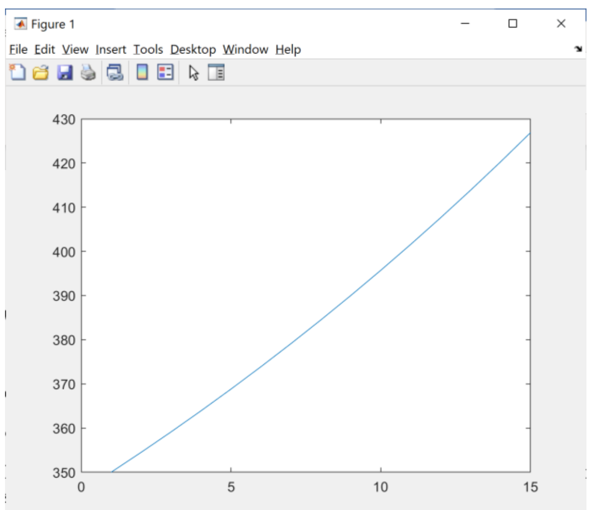
since each year the number of gannets grows at rate 3% and 6 of them will be hunted.

The initial condition is given by $g(1) = 350$

(b) We use the following commands in the command window. (You may not have the >> if you are writing a script instead of using the terminal)

```
>> g(1)=350;
>> for n = 2:15
g(n) = 1.03*g(n-1)-6;
end
>> plot(g)
```

The plot we obtain is



(c) The number of gannets will double, since initially the number is 350, and it will result in an increase of 350*3% = 10.5 gannets next year, which exceed the number of hunted gannets, which is only 6. Thus every year afterwards, the number of gannets will increase almost geometrically.

(d) The steady state is given by solving the equation

$$g = 1.03g - 6$$

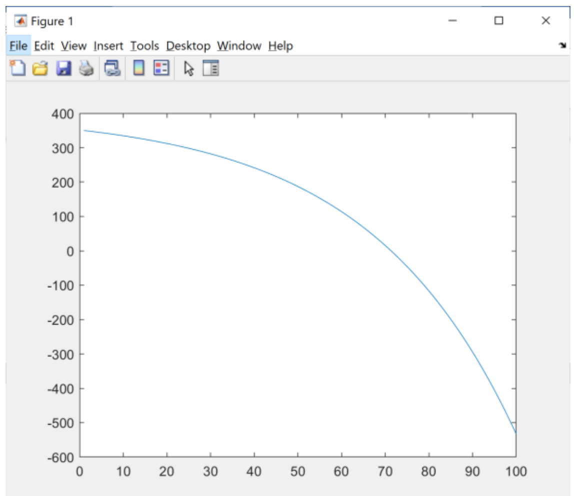
which gives $g = 200$. This is not a stable fixed point, as mentioned in (c), that if the number of gannets exceed this number, the number will keep increasing, while if lower than this number, the gannets will finally die out.

(e) We may solve this either numerically or analytically. Numerically the codes are

```
>> g(1)=350;
```

```
>> for n = 2:100
g(n) = 1.03*g(n-1)-12;
end
>> plot(g)
```

From the plot, we see the species die out near the 70-th year.



Analytically, we may solve out the difference equation to get

$$g(n) = 400 - 2^{3-2n} \times 25^{2-n} \times 103^{n-1}$$

Let $g(n) < 0$, we get $n > 70$, which is the same as what we have above.

(f) One way it is unrealistic is that when there are too many gannets, even if human beings don't hunt them, their number will stop to grow due to limited resources, and to remedy this we may introduce a factor that stops the species from reproducing after its total number reach some threshold value N. Another unrealistic simplification is that when the species is in an endangered state (too few of them), there is no way that human beings still hunt 12 of them per year, and that's also causing the $g(n)$ to become negative (which is impossible) in the long run. We may truncate the number of hunted gannets once the species has a population lower than certain value, as a protection.

5.

(a) For both γ there is one steady state near $x^* = -0.2$, simply from this plot it is hard to tell whether they are stable or not. It depends on the slope of the curve at the intersection. If $|f'(x^*)| > 1$ it is unstable, otherwise it is stable. In the second plot, we see that at the same stead state the two curves both have slope greater than 1. Now using the chain rule, we have

$$\left(f\big(f(x)\big)\right)' = f'\big(f(x)\big)f'(x) > 1$$

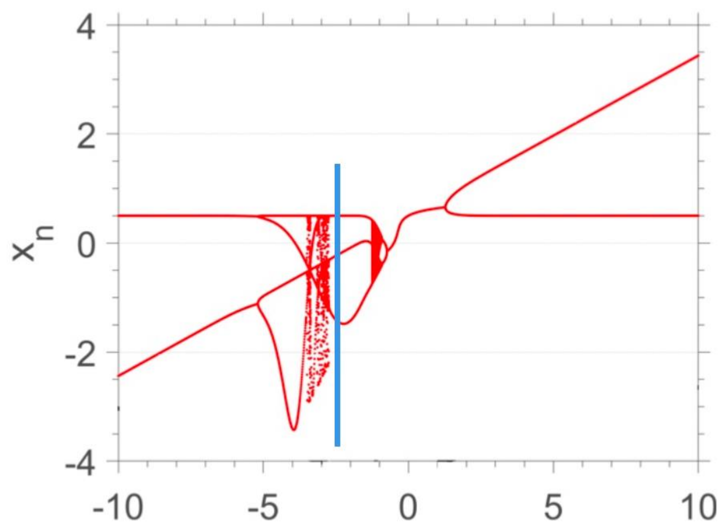
At $x = x^*$, we have $x^* = f(x^*)$ and so

$$f'\big(f(x^*)\big)f'(x^*) = \big(f'(x^*)\big)^2 > 1$$

which shows $|f'(x^*)| > 1$. Thus they are both unstable.

(b) For both γ, two other intersections appear and they should be 2-cycles. The steady state near $x = -0.2$ will not be of length 2 because they already appear in the first plot. They stability, again, as in (a), is hard to tell without analytic information. Still, eyeballing the plot can tell us that the two steady states of the blue curve should both be stable (abs of slope of the curve are less than 1), but for the red one it is hard to tell. If we use the bifurcation plot, we may say that they are all stable, since conventionally only stable fixed points are plotted in the bifurcation diagram (some people also plot the unstable ones, using dashed lines).

(c) Plot (I) is a 3-cycle, and according to the bifurcation diagram it should be somewhere in the blue line's region

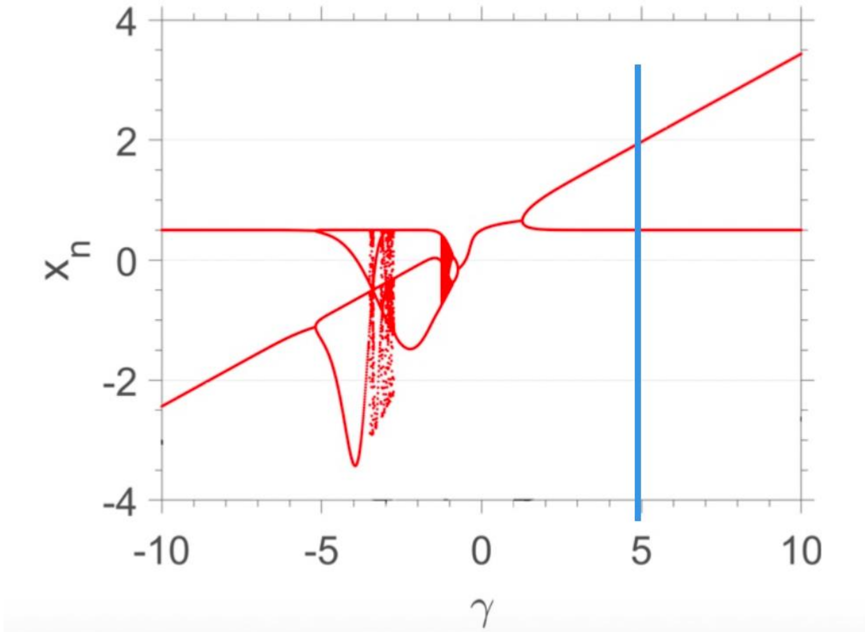


The only possibility is $\gamma = -1.5$.

Plot (II) is a 2-cycle, and the two steady points are near 0 and -0.3, thus the only possibility is $\gamma = -0.85$.

Plot (III) is the onset of chaos, and should correspond to $\gamma = -1.05$

(d) At $\gamma = 5$, as indicated by the blue line below, the system has 2 fixed points near $x = 0.25$ and $x = 2$



Without further information we cannot determine the exact shape of the curve, but using the bifurcation diagram, a 2-cycle should appear like this. In the long run, the trajectory of the system will converge to the 2-cycle indicated by the thin black square.