a) In order for the rocket to escape the planet's gravitational pull and travel to infinity, it must be given a tangential velocity equal to or greater than the escape velocity of the planet at the rocket's current altitude. The escape velocity can be calculated using the following formula:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass of the planet, and R is the distance from the center of the planet to the rocket's orbit.

To find the required tangential impulse, we can use the following formula:

$$\Delta v = v_e - v_o$$

where Δv is the change in velocity needed to escape, and v_0 is the rocket's current tangential velocity. Therefore, the tangential impulse required is:

$$\Delta v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

Simplifying this expression gives:

$$\Delta v = \sqrt{\frac{GM}{R}} \left(\sqrt{2} - 1\right)$$

b) If the rocket is given a tangential impulse less than the value calculated in part (a), it will not be able to escape the planet's gravitational pull and will instead follow a trajectory that brings it back around the planet. If the impulse is just slightly less than the escape impulse, the rocket will follow a highly elliptical orbit that brings it close to the planet's surface on its closest approach. To find the tangential impulse required to cause the rocket to graze the back side of the planet, we need to calculate the minimum tangential velocity required to maintain a circular orbit at the planet's surface. This can be done using the formula:

$$v_{circ} = \sqrt{\frac{GM}{R_o}}$$

where R_0 is the radius of the planet. Therefore, the tangential impulse required to cause the rocket to graze the back side of the planet is:

$$\Delta v = v_{circ} - v_o = \sqrt{\frac{GM}{R_o}} - \sqrt{\frac{GM}{R}}$$

c) When the rocket is released at a distance D from the center of the planet with zero velocity, it will fall towards the planet under the influence of gravity. The velocity of the rocket relative to the planet at any point in its descent can be calculated using the conservation of energy:

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{D}$$

where v is the velocity of the rocket relative to the planet at a distance r from the planet's center. Solving for v gives:

$$v = \sqrt{\frac{2GM}{D} - \frac{2GM}{r}}$$

The instant that the rocket hits the surface of the planet, r will be equal to the radius of the planet R_0. Therefore, the velocity of the rocket relative to the planet at impact is:

$$v = \sqrt{\frac{2GM}{D} - \frac{2GM}{R_o}}$$

Note that this assumes that there is no atmosphere to slow down the rocket's descent. If there is an atmosphere, the velocity at impact will be lower due to air resistance.

a) The comet's trajectory around the star will be an ellipse, with the star at one of the foci. The distance of closest approach, or periastron distance r_p, is given by the following equation:

$$r_p = \frac{l^2}{GMm^2}\frac{1}{1+e}$$

where G is the gravitational constant, e is the eccentricity of the orbit, and l is the angular momentum of the comet. The eccentricity of the orbit can be calculated from the comet's initial velocity and angular momentum using the formula:

$$e = \sqrt{1 + \frac{2El^2}{GM^2m^3}}$$

where E is the total energy of the comet. If the comet starts at infinity, its initial energy is:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

where v is the initial velocity of the comet and r is its initial distance from the star.
b) The speed of the comet at periastron can be found using the conservation of energy:

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = E$$

where v_p is the velocity of the comet at periastron. Solving for v_p gives:

$$v_p = \sqrt{\frac{2}{m}\left(E + \frac{GMm}{r_p}\right)}$$

c) The gravitational potential energy of the comet at periastron can be found using the formula:

$$U_p = -\frac{GMm}{r_p}$$

where r_p is the periastron distance.

a) If the rocket's orbital radius is the same as that of the Earth, its period T can be calculated using Kepler's third law:

$$T = 2\pi\sqrt{\frac{a^3}{GM_E}}$$

where a is the orbital radius (equal to the radius of the Earth, R_E), and G is the gravitational constant. M_E is the mass of the Earth. Substituting the values gives:

$$T = 2\pi\sqrt{\frac{(6.371\times10^6)^3}{(6.674\times10^{-11})(5.972\times10^{24})}} \approx 5066\ \mathrm{s}$$

b) In geosynchronous orbit, the rocket's period will be equal to the rotation period of the Earth, which is about 23 hours and 56 minutes, or 86,164 seconds. Using Kepler's third law, we can solve for the radius r of the rocket's orbit:

$$\frac{T^2GM_E}{4\pi^2} = r^3$$

Substituting the values gives:

$$r = \sqrt[3]{\frac{(86164)^2(6.674\times10^{-11})(5.972\times10^{24})}{4\pi^2}} \approx 4.22\times10^7\ \mathrm{m}$$

c) When the rocket is given a small radial impulse, it will start to oscillate about its equilibrium position in the radial direction. This is a simple harmonic motion, with a period given by:

$$T_r = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass of the rocket and k is the effective spring constant of the oscillation. The spring constant can be approximated by:

$$k \approx \frac{GM_E}{r^3}$$

where r is the radius of the rocket's orbit. Substituting the values gives:

$$k \approx \frac{(6.674\times10^{-11})(5.972\times10^{24})}{(4.22\times10^7)^3} \approx 2.72\times10^{-7}\ \mathrm{N/m}$$

Therefore, the period of oscillation is:

$$T_r = 2\pi\sqrt{\frac{m}{k}} \approx 2\pi\sqrt{\frac{m}{2.72\times10^{-7}}} \approx 2426\sqrt{m}\ \mathrm{s}$$