Under the new measure P, we have $E_t(B_t)=B_t$. Using Itô's lemma, we can derive the differential of B_t as follows:

$$d\widetilde{B}_t = d\left(\frac{B_t}{S_t}\right) = \frac{dB_t}{S_t} - \frac{B_t}{S_t^2}dS_t.$$

Since pi_tB_t is a martingale under P, we have $d(pi_tB_t)=pi_tdB_t+pi_tB_tdlog(pi_t)=pi_tdB_t+pi_tB_tdpi_t/pi_t$. By the same argument, we have $d(pi_tS_t)=pi_tdS_t+pi_tS_tdlog(pi_t)=pi_tdS_t+pi_tS_tdpi_t/pi_t$.

Using these expressions and the fact that pi_tB_t and pi_tS_t are both martingales under P, we obtain

$$0 = \mathbb{E}_t(d(\pi_T\widetilde{B}_TS_T)) = \mathbb{E}_t(\pi_Td\widetilde{B}_tS_T + \pi_T\widetilde{B}_tdS_T + \pi_T\widetilde{B}_TS_T\frac{d\pi_T}{\pi_T}),$$

where we have used the fact that pi_t and S_t are both F_t-measurable. Rearranging terms and dividing both sides by pi_tS_t, we get

$$d\widetilde{B}_t = -\frac{\pi_tB_t}{\pi_tS_t}d\log(\pi_t) + \frac{1}{\pi_tS_t}d\widetilde{\mathbb{P}}\left(\frac{\pi_T\widetilde{B}_TS_T}{\pi_tS_t}\right).$$

Now, we use the fact that $xi_t=pi_t\ S_t/S_0$ is a Radon-Nikodym derivative between P and P, i.e., $xi_t=dP/dP$. Therefore, we have $dP(X)=xi_tdP(X)$ for any F_t-measurable random variable X. Applying this to the above equation, we get

$$d\widetilde{B}_t = -\frac{\pi_tB_t}{\pi_tS_t}d\log(\pi_t) + \frac{1}{S_0}d\mathbb{P}(\xi_T\widetilde{B}_T),$$

which expresses the differential of B_t under the new measure P.

To derive the implied differential of S_t under P, we first need to express S_t in terms of S_t and B_t. Recall that $B_t=B_t/S_t$, so we have:

$S_t =S_t/B_t$.

Taking the differential of both sides, we get:

$dS_t = d\ (S_t/B_t) = dS_t/B_t - S_t/B_t^2dB_t$.

Using the expression we derived for dB_t in the previous part, we can substitute and simplify to get:

$dS_t = 1/B_t[dS_t - S_t/B_t\ (\mu\ dt + \sigma\ dW_t^P)]$.

This is the implied differential of S_t under P.

To price the call option using the new measure P, we need to compute the expectation of the discounted payoff under this measure. The discounted payoff for a European call option with strike K and maturity T is given by $(S_t - K)^+$. Using the fact that $S_t = S_t/S_0$ is a martingale under P, we have:

$$\widetilde{\mathbb{E}}_t[(S_T-K)^+] = \widetilde{\mathbb{E}}_t[(S_0\widetilde{S}_T-K)^+] = S_0\widetilde{\mathbb{E}}_t[(\widetilde{S}_T-\frac{K}{S_0})^+]$$

where E_t denotes the expectation under P.
To compute this expectation, we need to determine the distribution of S_t under P. From the Black-Scholes formula derived earlier, we have:

$$\ln \widetilde{S}_T = \ln \widetilde{S}_t + \left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}\widetilde{Z}$$

where Z is a standard normal random variable under P. Taking the exponential of both sides, we get:

$$\widetilde{S}_T = \widetilde{S}_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)(T-t) + \sigma\sqrt{T-t}\widetilde{Z}\right)$$

This shows that S_t is log-normally distributed under P with parameters $\ln S_t + (r - \sigma^2/2)(T-t)$ and σ*sqrt(T-t).
Let δ denote the cumulative distribution function of the standard normal distribution.
Thus, the price of the call option under P is:

$$\widetilde{C}_t = \widetilde{\mathbb{E}}_t\left[e^{-r(T-t)}(S_T - K)^+\right] = e^{-r(T-t)}\widetilde{\mathbb{E}}_t\left[(\widetilde{S}_T e^{r(T-t)} - K)^+\right]$$

where we have used the fact that $S_t = S_t / S_t$ and E_t denotes the expectation under P.
Substituting the expression for S_t and using the fact that $xi_t = pi_t\ S_t/S_0$, we need to determine the distribution of xi_t. Note that $xi_t = pi_t\ S_t/S_0$, and using the fact that $pi_t\ S_t = 1$.
Since $xi_t = pi_t/pi_t\ .\ xi_t$ and xi_t is a known function of t and the stock price, we only need to determine the distribution of pi_t / pi_t. Using the fact that $pi_t\ B_t$ is a martingale under P.
The expectation inside the parentheses can be computed using the cumulative distribution function (CDF) of the standard normal distribution evaluated at d_1 and d_2.
where N is the CDF of the standard normal distribution and d_1 and d_2 are defined as before:
$d_1 = [\ln(S_t/K) + (r + \sigma^2/2)(T-t)]/[\sigma$*sqrt(T-t)], $d_2 = d_1 - \sigma$*sqrt(T-t).
Therefore, the price of the call option under the new measure P is:
$C_t = S_t N(d_1) - K\ e^{-r(T-t)} N(d_2)$.
This is the Black-Scholes pricing formula under the new measure P using S as the numeraire.

Under the new measure P, the delta can be interpreted as the hedge ratio between the call option and the underlying asset, where the underlying asset is now measured in units of S instead of the original currency unit. Specifically, the delta $N(d_1)$ represents the amount of the underlying asset in units of S that should be held for every unit of the call option to perfectly hedge against small changes in the underlying asset price.
More formally, the delta is defined as the partial derivative of the call option price with respect to the underlying asset price, when the underlying asset is measured in units of S. Using the Black-Scholes formula derived under the new measure P, we have:
$\Delta = \partial C/\partial S = N(d_1)\ \partial d_1/\partial S = N(d_1)\ 1/[S^*\sigma^*\mathrm{sqrt}(T-t)]$
This interpretation of delta as the hedge ratio in units of S can be useful in situations where the underlying asset is subject to significant inflation or currency fluctuations, as using S as the numeraire allows for more stable pricing and hedging in such environments.