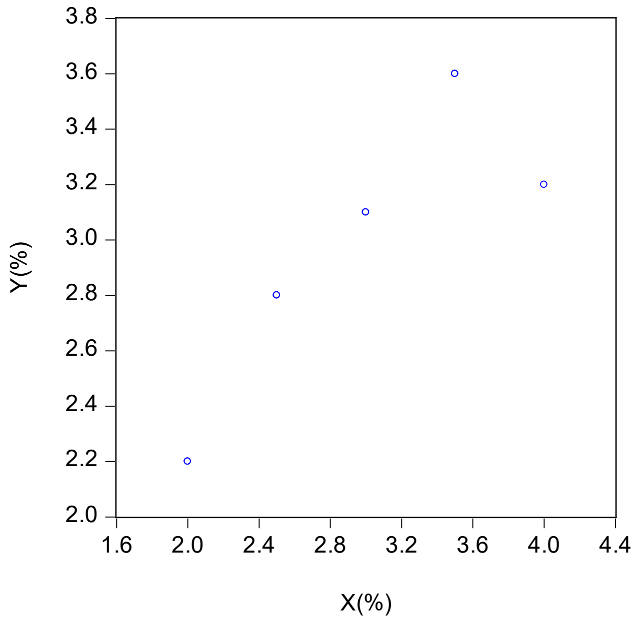
a)



linear regression model : $Y = \beta_0 + \beta_1 X + \varepsilon$

Assumptions :

1) The dependent variable Y has a linear relationship with the independent variable X.
2) X is not random.

b)

$$\hat{\beta}_1 = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} X_i^{\,2} - \left(\sum_{i=1}^{n} X_i\right)^2} = \frac{7}{12.5} = 0.56$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X} = 2.98 - 0.56 \times 3 = 1.3$$

Therefore, the estimated regression line is $\hat{Y} = 1.3 + 0.56X$

c)

The $\hat{\beta}_1$ means that for every 1 unit change in X, Y changes by an average of 0.56 units

d)

ANOVA Table

	df	SS	MS	F	Significance F
Regression	1	0.784	0.784	7.736842105	0.068903509
Error	3	0.304	0.101333333		
Total	4	1.088			

From the ***ANOVA*** table we know that the ***Prob(F-statistic)=0.068904<10%***, so the regression equation has passed F-test and the equation is significant.

e)

For the slope, we hypothesize that

$$H_0 : \beta_1 = 0$$

$$H_1 : \beta_1 \neq 0$$

We do the t-test:

$$s_e = \sqrt{\frac{\sum (Y_i - \hat{Y}_i)^2}{n-2}} = \sqrt{\frac{0.304}{3}} = 0.318329$$

$$s_{\hat{\beta}_1} = \frac{s_e}{\sqrt{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2}} = \frac{0.3183289}{\sqrt{2.5}} = 0.201329$$

$$t = \frac{\hat{\beta}_1}{s_{\hat{\beta}_1}} = \frac{0.56}{0.201329} = 2.7815$$

At the 2% significance level, the critical value $t_{0.02/2}$ is 4.5407. Because $t=2.7815<t_{0.02/2}$, we do not reject the H_0. Therefore, the slope are not different from zero.

For the intercept, we hypothesize that

$$H_0 : \beta_0 = 0$$

$$H_1 : \beta_0 \neq 0$$

$$s_{\hat{\beta}_0} = s_e \sqrt{\frac{1}{n} + \frac{\overline{X}}{\sum_{i=1}^{n} (X_i - \overline{X})^2}} = 0.620537$$

$$t = \frac{\hat{\beta}_0}{s_{\hat{\beta}_0}} = \frac{1.3}{0.620537} = 2.09496$$

Because $t=2.09496<t_{0.02/2}$, we do not reject the H_0. Therefore, the intercept are not different from zero.

f)

When X=1%,

$$\hat{Y} = 1.3 + 0.56 \times 1 = 1.86$$

And the prediction interval of Y is

$$\hat{Y} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{\sum_{i=1}^{n} (X_i - \overline{X})^2}} = 1.86 \pm 3.1824 \times 0.318329 \times \sqrt{2.8} = 1.86 \pm 1.695157$$

$$0.1648 \le \hat{Y} \le 3.5552$$

I think the interval is wide, 1.695157 is almost equal to 1.86, so the point forecast is not reliable.

g)

$$s_{ind} = s_e \sqrt{1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{\sum_{i=1}^{n} (X_i - \overline{X})^2}} = 0.318329$$

$$\because Y \sim N(1.86, 0.10133)$$

$$\therefore \frac{Y - 1.86}{0.318329} \sim N(0,1)$$

$$\therefore P(1 \le Y \le 2) = P\left(-2.7016 \le \frac{Y - 1.86}{0.318329} \le 0.4398\right) = \Phi(0.4398) - \Phi(-2.7016) \approx 66.65\%$$

The probability that the inflation range of 1% and 2% can be achieved is 66.65%. Therefore, it is very likely to be achieved.

h)

$$s_e^2 = \frac{\sum (Y_i - \hat{Y}_i)^2}{n-2} = 0.1013$$

$$\because \frac{(n-2)s^2}{\sigma^2} \sim \chi^2(n-2)$$

$$\therefore \chi_{1-\alpha/2}^2 \le \frac{(n-2)s^2}{\sigma^2} \le \chi_{\alpha/2}^2$$

$$\therefore \frac{(n-2)s^2}{\chi_{\alpha/2}^2} \le \sigma^2 \le \frac{(n-2)s^2}{\chi_{1-\alpha/2}^2}$$

The 95% confidence level interval is

$$0.05897 \le \sigma \le 2.5522$$