(a)

1.There should be a linear and additive relationship between dependent (response) variable and independent (predictor) variables.

2. There should be no correlation between the error terms.

3. The independent variables should not be correlated.

4. The variance of the error term is the same for all observations.

5. The error terms must be normally distributed.

6.The expected value of the error terms of the independent variable is 0.

(b)

$$Y = X\beta + \varepsilon.$$

$$Q = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon'\varepsilon = (Y - X\hat{\beta})'(Y - X\hat{\beta})$$

$\because$ we know that $(AB)' = B'A'$

$\therefore$ $(Y - X\hat{\beta})'(Y - X\hat{\beta}) = Y'Y - \hat{\beta}'X'Y - Y'X\hat{\beta} + \hat{\beta}'X'X\hat{\beta}$

take partial derivative with $\hat{\beta}$

$$\frac{\partial Y'Y}{\partial \beta} = 0 \qquad \frac{\partial \hat{\beta}'X'Y}{\partial \hat{\beta}} = X'Y \qquad \frac{\partial Y'X\hat{\beta}}{\partial \hat{\beta}} = (Y'X)' = X'Y$$

$$\frac{\partial \hat{\beta}'X'X\hat{\beta}}{\partial \beta} = 2X'X\hat{\beta}$$

So $-X'Y - X'Y + 2X'X\hat{\beta} = 0$

$$(X'X)\hat{\beta} = X'Y$$

$$\hat{\beta} = (X'X)^{-1}X'Y$$

(c)

$$Var(\hat{\beta}) = E\{[\hat{\beta}-E(\hat{\beta})][\hat{\beta}-E(\hat{\beta})]'\}$$

$$= E[(\hat{\beta}-\beta)(\hat{\beta}-\beta)']$$

$$= E[(X'X)^{-1}X'\varepsilon\varepsilon'X(X'X)^{-1}]$$

$$= (X'X)^{-1}X'E(\varepsilon\varepsilon')X(X'X)^{-1}$$

$$= (X'X)^{-1}X'\sigma^2 I_n X(X'X)^{-1}$$

$$= \sigma^2 (X'X)^{-1}$$

(d)

①

Let $\Omega = DD'$

$\because \; Y = X\beta + \varepsilon$

$\therefore \; D^{-1}Y = D^{-1}X\beta + D^{-1}\varepsilon$

Let $D^{-1}Y = Y^*$

$D^{-1}X = X^*$

$D^{-1}\varepsilon = \varepsilon^*$

$$Y^* = X^*\beta + \varepsilon^*$$

$$Var(\varepsilon^*) = E(\varepsilon^*\varepsilon^{*\prime}) = E(D^{-1}\varepsilon\varepsilon'D^{-1})$$

$$= D^{-1}E(\varepsilon\varepsilon')D^{-1}$$

$$= D^{-1}\sigma^2\Omega D^{-1} = \sigma^2 D^{-1}DD'D^{-1}$$

$$= \sigma^2 I$$

because of OLS, we know that (in question (b))

$$\hat{\beta} = (X^{*\prime}X^*)^{-1}X^{*\prime}Y^*$$

$$= (X'D^{-1\prime}D^{-1}X)^{-1}X'D^{-1\prime}D^{-1}Y$$

$$= (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y$$

$\therefore \; \hat{\beta} = (X'X)^{-1}X'Y$ is not an unbiased estimator of β.

②

$$Var(\hat{\beta}) = E[(\hat{\beta}-\beta)(\hat{\beta}-\beta)']$$

$$= E[(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}\varepsilon\varepsilon'\Omega^{-1}X(X'\Omega^{-1}X)^{-1}]$$

$$= \sigma^2(X'\Omega^{-1}X)^{-1}$$