

Since $\mathcal{F}_t$ is a filtration, we have the following inclusion of σ -algebra that for $s \leq t$, we have

$$\mathcal{F}_s \subset \mathcal{F}_t$$

Using the tower property of conditional expectation, we have

$$\mathbb{E}(X_t|\mathcal{F}_s) = \mathbb{E}(\mathbb{E}(X|\mathcal{F}_t)|\mathcal{F}_s) = \mathbb{E}(X|\mathcal{F}_s) = X_s$$

On the other hand, using the law of total expectation, we have

$$\mathbb{E}(|X_t|) = \mathbb{E}(|\mathbb{E}(X|\mathcal{F}_t)|) \leq \mathbb{E}(|X|) < \infty$$

The two properties combine to show that X_t is a martingale.