(a) Proof: Since nullity(T) = 0, it follows that T is one-to-one, and thus T has a left inverse T^(-1) : W → V, such that T^(-1) T = I_V. Then, we have:

nullity (T o S) = dim (ker (T o S)) = dim ({u ∈ U : (T o S)(u) = 0V})
= dim ({u ∈ U : S(u) ∈ ker T}) = dim (ker S).

Since the dimension of a subspace is invariant under bijective linear transformations, it follows that nullity (T o S) = nullity (S).

(b) Proof: The rank-nullity theorem states that for any linear transformation S: U → V, we have:

rank S + nullity S = dim U.

Applying this to T o S and S, we have:

rank (T o S) + nullity (T o S) = dim U = rank S + nullity S.

Since nullity (T o S) = nullity (S), it follows that rank (T o S) = rank (S).