For a potential well between 0 and a, the eigenfunctions are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

For a potential well between $-\frac{a}{2}$ and $\frac{a}{2}$, the eigenfunctions are given by

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right), & \text{even } n \\ \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi}{a}x\right), & \text{odd } n \end{cases}$$

In order to go from the second set of solutions to the first set of solutions, notice that if we let $x' = x + \frac{a}{2}$, then $x = 0$ will be shifted to $x' = \frac{a}{2}$, which means that a wave-function defined on $\left[-\frac{a}{2}, \frac{a}{2}\right]$ will be shifted to a wave-function defined on $[0, a]$. We put $x' = x + \frac{a}{2}$ into the second set of solutions to get

$$\psi_n(x') = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\left(x' - \frac{a}{2}\right)\right), & \text{even } n \\ \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi}{a}\left(x' - \frac{a}{2}\right)\right), & \text{odd } n \end{cases}$$

For even n, we have

$$\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\left(x' - \frac{a}{2}\right)\right) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x' - \frac{n\pi}{2}\right) = (-1)^{n/2}\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x'\right)$$

For odd n, we have

$$\sqrt{\frac{2}{a}}\cos\left(\frac{n\pi}{a}\left(x' - \frac{a}{2}\right)\right) = \sqrt{\frac{2}{a}}\cos\left(\frac{n\pi}{a}x' - \frac{n\pi}{2}\right) = (-1)^{(n-1)/2}\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x'\right)$$

We see that no matter n is even or odd. The eigenfunction after transformation is always a sine function. Though there may be a change of sign, but quantum mechanics do not care about a phase factor.