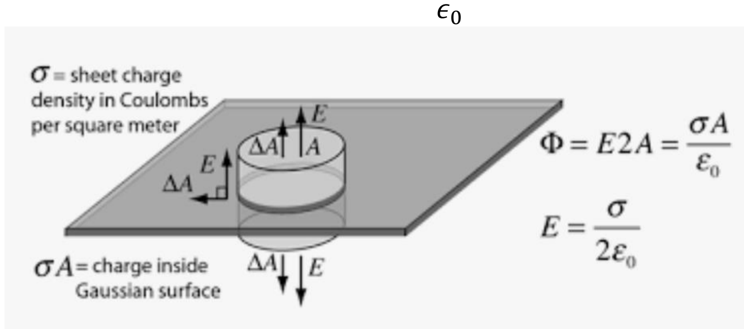
1. (i)
   We use the Gauss's law by choosing an infinitesimal cylinder to be our Gaussian surface which has its disk faces parallel to the infinitesimal sheet (since the sheet is infinitesimal, we may regard it to be planar).

   Then, due to symmetry, we know that the electric field must be perpendicular to the sheet (for otherwise we may rotate the system to obtain a contradiction). Set the strength of the field to be E. By Gauss's law, the flux is given by
$$\Phi = 2AE = \frac{A\sigma}{\epsilon_0}$$


   where A is the area of the disk face of the cylinder, and $A\sigma$ gives the total charge enclosed. We then have
$$E = \frac{\sigma}{2\epsilon_0}$$
   Notice that E is perpendicular to the sheet and points away from the sheet (since the flux is positive), so if we go from one side of the sheet to the other, the electric field will change its direction abruptly at the sheet, causing a discontinuity. The field component changes by
$$2E = \frac{\sigma}{\epsilon_0}$$
   (ii) As the result above shows that the electric field only depends on σ, which is taken to be constant over the sheet since it is uniformly charged. The perpendicular component is always $E = \sigma/2\epsilon_0$ and thus continuous across the sheet.

2. Setting up a polar coordinate, by Coulomb's law, an infinitesimal piece of the disk at (r, θ) will contribute to the potential by
$$dV = \frac{\sigma}{4\pi\epsilon_0\sqrt{r^2 + z^2}} r dr d\theta$$
   The potential is given by the following integral
$$V(z) = \int_0^R dr \int_0^{2\pi} \frac{\sigma}{4\pi\epsilon_0\sqrt{r^2 + z^2}} r dr d\theta$$
$$= \frac{\sigma}{2\epsilon_0}\int_0^R \frac{r}{\sqrt{r^2 + z^2}} dr = \frac{\sigma}{2\epsilon_0}\left[\sqrt{r^2 + z^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}(\sqrt{R^2 + z^2} - z)$$
3. (i)
   Due to symmetry, the field along the axis of symmetry will be perpendicular to the disk. The field is given by taking derivative with respect to z and adding a minus sign, which gives
$$E(z) = -\frac{d}{dz}V(z) = -\frac{\sigma}{2\epsilon_0}\left(\frac{z}{\sqrt{R^2 + z^2}} - 1\right)$$

where the minus sign indicates the direction of the field (positive if pointing away from the disk, and negative if towards the disk). We see that $E(0) = \sigma/2\epsilon_0$ at $z = 0$, which gives back the same discontinuity which we have already obtained in problem 1.

(ii) For $|z| \gg R$, we have

$$\frac{z}{\sqrt{R^2 + z^2}} = \pm \frac{1}{\sqrt{1 + \left(\frac{R}{z}\right)^2}} \to \pm \left(1 - \frac{1}{2}\left(\frac{R}{z}\right)^2\right)$$

where the sign $\pm$ depends on the sign of z. Without loss of generality we take it to be positive. It is obtained by Taylor expanding the infinitesimal term (R/z).
Put back to get

$$E(z) \to -\frac{\sigma}{2\epsilon_0}\left(\left(1 - \frac{1}{2}\left(\frac{R}{z}\right)^2\right) - 1\right) = \frac{\sigma R^2}{4\epsilon_0 z^2} = \frac{\sigma \pi R^2}{4\pi\epsilon_0 z^2}$$

Notice that this is the same as the field produced by a point charge of $\sigma\pi R^2$.

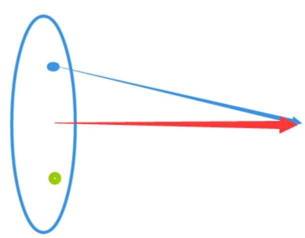
(iii) If we add a higher order term in the expansion above, we get

$$\frac{1}{\sqrt{1 + \left(\frac{R}{z}\right)^2}} \to 1 - \frac{1}{2}\left(\frac{R}{z}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}\left(\frac{R}{z}\right)^4$$

Put back to get

$$E(z) \to -\frac{\sigma}{2\epsilon_0}\left(1 - \frac{1}{2}\left(\frac{R}{z}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}\left(\frac{R}{z}\right)^4 - 1\right)$$
$$= \frac{\sigma}{2\epsilon_0}\left(\frac{1}{2}\left(\frac{R}{z}\right)^2 - \frac{1}{8}\left(\frac{R}{z}\right)^4\right)$$

We see the the correction term is negative, meaning that the field is smaller than that produced by a point charge. It makes since because, as shown in the diagram below



a general point on the disk (not at the center) is farther away to our target point than the point at the center, so the blue arrow is in general longer than the red one, so when we use Coulomb's law, since distance square goes to the denominator, the actually electric field is smaller than our crude single-point-charge approximation. On the other hand, the electric field produced by an infinitesimal area near the blue point can only contribute a component perpendicular to the disk since the component parallel to the disk plane will be cancelled with that produced by its antipole (the green one). Thus, as we add another correction term, the final result should decrease, which explains the minus sign.