(a)

Let $Z \sim N(0, t)$

Then $\frac{Z}{\sqrt{t}} \sim N(0, 1)$

By problem 1.10,

$E\left(\frac{Z}{\sqrt{t}}\right)^6 = 5 \times 3 \times 1 = 15$

$E Z^6 = 15 t^3$

Since B_t is a Brownian motion

$B_t \sim N(0, t)$

Then

$E B_t^6 = 15 t^3$

(b)

By the definition of Brownian motion

$B_{t_2} - B_{t_1}$ and $B_{t_3} - B_{t_2}$ are independent.

Then

$$E[(B_{t_2} - B_{t_1})(B_{t_3} - B_{t_2})]$$

$$= E(B_{t_2} - B_{t_1})\, E(B_{t_3} - B_{t_2})$$

$$= 0$$

(c)

$E\, B_s^2 B_t^2$

$= E\, B_s^2 (B_t - B_s + B_s)^2$

$= E[\, B_s^2 (B_t - B_s)^2 + 2 B_s^3 (B_t - B_s) + B_s^4]$

$= E\, B_s^2 (B_t - B_s)^2 + 2 E\, B_s^3 (B_t - B_s) + E\, B_s^4$

$= E\, B_s^2 \; E(B_t - B_s)^2 + 2 E\, B_s^3 \; E(B_t - B_s) + E\, B_s^4$

$= s \cdot (t - s) + 0 + 3 s^2$

$= ts + 2 s^2$

(d)

$E(B_s B_t^3)$

$= E\left[B_s (B_t - B_s + B_s)^3\right]$

$= E\, B_s (B_t - B_s)^3 + 3 E\, B_s^2 (B_t - B_s)^2$

$+ 3 E\, B_s^3 (B_t - B_s)$

$+ E\, B_s^4$

$= E B_s\, E(B_t - B_s)^3 + 3 E B_s^2\, E(B_t - B_s)^2$

$+ 3 E B_s^3\, E(B_t - B_s) + E B_s^4$

$= 0 + 3 \cdot s \cdot (t-s) + 0 + 3 s^2$

$= 3ts - 3s^2 + 3s^2$

$= 3ts$