(a)

$$E(Z g(Z))$$

$$= \int_{-\infty}^{\infty} Z g(Z) e^{-\frac{Z^2}{2}} \frac{1}{\sqrt{2\pi}} dZ$$

$$= - \int_{-\infty}^{\infty} g(Z) \frac{1}{\sqrt{2\pi}} d e^{-\frac{Z^2}{2}}$$

Integration by parts

$$= - \left[\underbrace{g(Z) e^{-\frac{Z^2}{2}} \Big|_{-\infty}^{\infty}}_{\text{this is 0 by assumption}} - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} d g(Z) \right]$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} d g(Z)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} g'(Z) dZ$$

$$= E(g'(Z))$$

(b)

Let $g(z) = z^{2n-1}$

$g'(z) = (2n-1) z^{2n-2}$

So

$E(z^{2n}) = E(z g(z))$

$= E g'(z)$

$= (2n-1) E(z^{2n-2})$

Let $a_n = E z^{2n}$

Then $a_n = (2n-1) a_{n-1}$

$= (2n-1)(2n-3) a_{n-2}$

$= \cdots$

$= (2n-1)(2n-3) \cdots 5 \cdot 3 \cdot 1$

$= \dfrac{(2n)!}{2^n \cdot n!}$