(a)

Model 1. C

$$X_t = Z_t + 0.7 Z_{t-1}$$

MA(1) process with $\theta = 0.7$, this process has positive ACF with a large value at lag one. This makes the series smooth with relatively high frequency components.

Model II B

AR(1) process with $\phi = -0.8 < 0$.

Thus $\rho(1) < 0$.

Thus the spectral density rapidly about its mean.

Model III A

This is a seasonal model with $s=6$

Thus we choose A

Model IV D

For $\cos\frac{\pi t}{4}$ term, the period is $s=\frac{2\pi}{\pi/4}$

$=8$

For $\cos\frac{\pi t}{2}$ term, the period is $s=\frac{2\pi}{\pi/2}$

$=4$

(b)

The shape of the spectral estimate will change

$$X_t = 0.9 X_{t-6} + Z_t$$

$$= 0.9(0.9 X_{t-12} + Z_{t-6}) + Z_t$$

$$= 0.81 X_{t-12} + 0.9 Z_{t-6} + Z_t$$

Let $\tilde{Z}_t = 0.9 Z_{t-6} + Z_t$

with lag 12, $\tilde{Z}_t$ and $\tilde{Z}_{t+12}$ are not correlated.

Thus

$$X_t = 0.81 X_{t-12} + \tilde{Z}_t$$

when we observe data yearly, this process is AR(1).