(a) 1) symmetric:

$$< \vec{x}, \vec{y} >= \vec{y}^T A\vec{x} = \vec{x}^T A\vec{y} =< \vec{y}, \vec{x} > .$$

2) linearity

$$< a\vec{x} + b\vec{y}, \vec{z} >= (a\vec{x} + b\vec{y})^T A\vec{z} = a\vec{x}^T A\vec{z} + b\vec{y}^T A\vec{z} = a < \vec{x}, \vec{y} > +b < \vec{x}, \vec{z} >$$

3) Positive definite

$$< \vec{x}, \vec{x} >= \vec{x}^T A\vec{x} \geq 0,$$

since A is a positive definite symmetric matrix.

Thus, $<, >$ is an inner product.

b) Consider

$$A = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

and $\vec{x} = (1, 1)^T$. Then,

$$\vec{x}^T A\vec{x} = -2.$$