

$$|A - \lambda I| = (1 - \lambda)(2 - \lambda)^5 = 0$$

$$\lambda_1 = 1 \quad \lambda_2 = 2$$

First, we calculate the eigenvectors

For $\lambda_1 = 1$

$$A x = x$$

So

$$x_1 + x_2 + x_6 = x_1$$

$$2x_1 + x_4 + 2x_5 + 3x_6 = x_2$$

$$2x_3 + x_5 - x_6 = x_3$$

$$2x_4 + x_5 + 2x_6 = x_4$$

$$2x_5 = x_5$$

$$2x_6 = x_6$$

So $v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ is an eigenvector

For $\lambda_2 = 2$

$$A - 2I = \begin{bmatrix} -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

with reduced row form

$$\begin{bmatrix} -1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So

$$N(A - 2I) = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Denote

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

We solve

$$(A-2I)X = v_1$$

$$x_5 + 2x_6 = 0$$

$$x_5 - x_6 = 0$$

$$x_4 + 2x_5 + 3x_6 = 1$$

$$-x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

$$\Rightarrow \begin{cases} x_4 = 1 \\ -x_1 + x_2 + x_3 = 0 \end{cases}$$

$$\text{So } v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(A-2I)X = v_4$$

$$x_5 + 2x_6 = 1$$

$$x_5 - x_6 = 0$$

$$x_4 + 2x_5 + 3x_6 = 1$$

$$-x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$

$$v_4 = \begin{pmatrix} 1 \\ 0 \\ -\frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$(A - 2I) X = V_2$$

$$X_5 + 2X_6 = 0$$

$$X_5 - X_6 = 1$$

$$X_4 + 2X_5 + 3X_6 = 0$$

$$-X_1 + \cancel{2}X_2 + X_3 + X_4 + X_5 + X_6 = 1$$

$$V_5 = \begin{pmatrix} 1 \\ 0 \\ \cancel{0} \\ \frac{1}{3} \\ 2 \\ 3 \\ -\frac{1}{3} \end{pmatrix}$$

Take

$$P = (w_1, v_1, v_3, v_4, v_2, v_5)$$

$$J = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$