1. $\frac{8}{3}$ $\left[\frac{2}{3}, 0, \frac{1}{3}\right]$

The dual problem is

maximize $z = y_1 + 3y_2$

subject $y_1 + 3y_2 \leq 2$ (1)

$2y_1 + 3y_2 \leq 5$ (2)

$4y_1 + 3y_2 \leq 4$ (3)

$z y_1 + 3y_2 = 2 \times \frac{2}{3} + 3 \times \frac{4}{3}$

$= \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

The minimum value is $\frac{8}{3}$

(2) is not binding so $x_2 = 0$

Then minimize $2x_1 + 4x_3$

subject to $x_1 + 4x_3 \geq 2$

$x_1 + x_3 \geq 1$

So $x_1 = \left(\frac{2}{3}, \frac{1}{3}\right)$

2. $-\frac{82}{3}$

$\left\{\frac{14}{3}, 0, \frac{4}{3}\right\}$

max $6x_1 - 4x_2 + 2x_3$

$x_1 + 2x_2 + x_3 + s_1 = 6$

$x_1 - x_2 - 2x_3 + s_2 = 2$

$-x_1 - x_2 - 3x_3 + s_3 = -3$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

		6	-4	2				
C_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	b
0	s_1	1	2	1	1	0	0	6
0	s_2	(1)	-1	-2	0	1	0	2
0	s_3	-1	-1	-3	0	0	1	-3
		6	-4	2				
0	s_1	0	3	3 ✓	1	-1	0	4
6	x_1	1	-1	-2	0	1	0	2
0	s_3	0	-2	-5	0	1	1	-1
		0	2	14 ✓				

		6	-4	2				
2	x_3	0	1	1	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{4}{3}$
6	x_1	1	1	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{14}{3}$
0	s_3	0	3	0	$\frac{5}{3}$	$-\frac{2}{3}$	1	$\frac{17}{3}$

3. $-\frac{131}{3}$

$\left[\frac{2}{5}, 0, 4\right]$

we rewrite

maximize $-4x_1 - 8x_2 + 12x_3$

subject to $3x_1 + 2x_2 + x_3 + s_1 = 6$

$6x_1 + 3x_2 + 2x_3 + s_2 = 6$

$-3x_1 + x_2 + 2x_3 + s_3 = 6$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

		c_j	-4	-8	12	0	0	0	
C_B	X_B		x_1	x_2	x_3	s_1	s_2	s_3	b
0	s_1		3	2	1	1	0	0	6
0	s_2		6	3	-2	0	1	0	6
0	s_3		-3	1	2	0	0	1	6
			-4	-8	12				

C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	b
			-4	-8				
0	S_1	$\frac{1}{2}$	$\frac{3}{2}$	0	1	0	$-\frac{1}{2}$	3
0	S_2	3	4	0	0	1	1	12
12	X_3	$-\frac{3}{2}$	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	3

$-4 + 12 \times \frac{3}{2} = 14$

C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	b
		-4	-8	12				
-4	X_1	1	$\frac{1}{3}$	0	$\frac{2}{9}$	0	$-\frac{1}{9}$	$\frac{2}{3}$
0	S_2	0	1	0	$-\frac{2}{3}$	1	$\frac{4}{3}$	10
12	X_3	0	1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	4

$S_0 \quad (X_1, X_2, X_3) = (\frac{2}{3}, 0, 4)$