

$$(a) \forall f=ax+b, g=cx+d \in S.$$

$$f \cdot g = f(g(x)) = f(cx+d) = a(cx+d)+b = acx+ad+b \in S.$$

$$g \cdot f = g(f(x)) = g(ax+b) = c(ax+b)+d = acx+bc+d \in S$$

$\Rightarrow (S, \cdot)$  is closed.

$$(b) e=x \in S.$$

$$\forall f=ax+b, f \cdot e = f(e(x)) = f(x) = ax+b$$

$$e \cdot f = e(f(x)) = f(x) = ax+b$$

$\Rightarrow e$  is the identity element of  $(S, \cdot)$

$$(c) \text{ let } f=1, g=x+1.$$

$$\Rightarrow \begin{cases} f \cdot g = 1 \\ g \cdot f = 2 \end{cases} \Rightarrow f \cdot g \neq g \cdot f$$

So  $(S, \cdot)$  is not a group

$$(d) S' = \{f \mid f(x) = ax, a \neq 0, a, x \in \mathbb{R}\}$$

$$(1) \forall f=ax, g=bx \in S', f \cdot g = f(g(x)) = abx \in S'$$

$$(2) \forall f=ax, g=bx, h=cx \in S'$$

$$(f \cdot g) \cdot h = (f \cdot g)(h(x)) = f(g(h(x))) = abcx$$

$$f \cdot (g \cdot h) = f \cdot g(h(x)) = f(g(h(x))) = abcx = (f \cdot g) \cdot h.$$

$$(3) e=x, \forall f=ax \in S'.$$

$$e \cdot f = e(f(x)) = ax = f(e(x)) = f \cdot e$$

$$(4) \forall f=ax \in S', \exists \frac{1}{a} \in \mathbb{R}, \text{ s.t. } f' = \frac{1}{a}x.$$

$$\Rightarrow f \cdot f' = f(f'(x)) = x = f'(f(x)) = f' \cdot f = e = x.$$

So  $(S', \cdot)$  is a group.