proof: (a) $C_1 \supset C_2 \supset C_3 \supset \cdots \supset C_k \supset \cdots$

Thus. $m(C) = m(\bigcap_{k=1}^{\infty} C_k) = \lim_{k\to\infty} m(C_k) = \lim_{k\to\infty} (1 - \sum_{i=1}^{k} 2^{i-1} l_i)$.

And. $\sum_{k=1}^{\infty} 2^{k-1} l_k < 1$.

So. $\lim_{k\to\infty} (1 - \sum_{i=1}^{k} 2^{i-1} l_i) = 1 - \sum_{k=1}^{\infty} 2^{k-1} l_k$.

$\Rightarrow$ $m(C) = 1 - \sum_{k=1}^{\infty} 2^{k-1} l_k$.

(b) C is perfect $\Leftrightarrow$ $C = C'$

Since every set C_k is a non-empty bounded closed set, and $C_k \supset C_{k+1}$ $(k \in \mathbb{N})$, the set $C = \bigcap_{k=1}^{\infty} C_k$ is a non-empty bounded closed set according to Cantor's bounded closed interval set theorem. $\Rightarrow C' \subset C$.

If $x_0 \in C$, then. $x_0 \in C_k$ $(k = 1, 2, \cdots)$.

$\forall \delta > 0, \exists k \in \mathbb{N}$, s.t. $l_k < \delta$. $\Rightarrow (x_0 - \delta, x_0 + \delta) \subset C_k$.

So x_0 is the limit point of C. $\Rightarrow C \subset C'$. $\Rightarrow C = C'$

So. C is perfect.

(c) let. $x_0 \in C$., for all $(x_0 - \delta, x_0 + \delta)$, $\delta > 0$. and is small enough let $l_k < \delta$.

$\Rightarrow (x_0 - \delta, x_0 + \delta) \subset C_k$.

But at the $k+1^{th}$ - stage, some elements of $(x_0 - \delta, x_0 + \delta)$ will be removed. So. $(x_0 - \delta, x_0 + \delta) \not\subset C_{k+1}$.

$\Rightarrow (x_0 - \delta, x_0 + \delta) \not\subset C$.

So C is totally disconneeted.

d) If $\sum_{k=1}^{\infty} 2^{k-1} l_k < 1$, then from (a) we know that $m(C) = 1 - \sum_{k=1}^{\infty} 2^{k-1} l_k$. $\Rightarrow C$ is uncountable.

If $\sum_{k=1}^{\infty} 2^{k-1} l_k = 1$, then $\exists k_0 \in \mathbb{N}$, s.t. $l_k = q^k$, $k > k_0$, $q \in (0,1)$

In this case, we can expand the numbers in set $[0,1]$ by $\frac{1}{q}$ decimal places.

So, the elements x of C is going correspond to $\sum_{n=1}^{\infty} a_n q^n$ $(a_n = 0, 1, \cdots, \frac{1}{q} - 1)$.

$\Rightarrow C \sim [0,1] \Rightarrow C$ is uncountable.