$$EY_i = EX_i^{C_i}$$

$$= E(X_i^{C_i} \mid C_i = 1)\, p(C_i = 1)$$

$$+ E(X_i^{C_i} \mid C_i = 2)\, p(C_i = 2)$$

$$= \frac{1}{2} E(X_i \mid C_i = 1) + \frac{1}{2} E(X_i^2 \mid C_i = 2)$$

$$= \frac{1}{2}\theta + \frac{1}{2}\theta = \theta$$

$$EY_i^2 = EX_i^{2C_i}$$

$$= \frac{1}{2} E(X_i^{2C_i} \mid C_i = 1) + \frac{1}{2} E(X_i^{2C_i} \mid C_i = 2)$$

$$= \frac{1}{2} E(X_i^2 \mid C_i = 1) + \frac{1}{2} E(X_i^4 \mid C_i = 2)$$

$$= \frac{1}{2}(\theta^2 + 1) + \frac{1}{2} \cdot 3 \cdot \theta^2$$

$$= 2\theta^2 + \frac{1}{2}.$$

$$\operatorname{var} Y_i = EY_i^2 - (EY_i)^2$$

$$= 2\theta^2 + \frac{1}{2} - \theta^2 = \theta^2 + \frac{1}{2}.$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

By the CLT, if n is large

$$\bar{Y} - E\bar{Y} \overset{\sim}{\sim} N(0, \operatorname{var}\bar{Y})$$

Note that $E\bar{Y} = \theta$

$$\operatorname{var}\bar{Y} = \frac{\operatorname{var}Y_i}{n} = \frac{\theta^2 + \frac{1}{2}}{n}$$

we can estimate $\operatorname{var}\bar{Y}$ by $\frac{\bar{Y}^2 + \frac{1}{2}}{n}$.

Then an approximate $(1-\alpha)$ CI for θ is

$$\bar{Y} \pm z_{1-\alpha/2}\sqrt{\frac{\bar{Y}^2 + \frac{1}{2}}{n}}.$$

So $g(\bar{Y}) = \sqrt{\frac{\bar{Y}^2 + \frac{1}{2}}{n}}$