

$$\begin{array}{c} \text{---} A \text{---} \xrightarrow{\tau \rightarrow \infty} \\ \uparrow \\ 0 \end{array} \quad \lim_{A \rightarrow \infty} U(A, t) = 0$$

$$T(A=0, t) = b \sin(\omega t)$$

$$U(A, t) = \bar{U}(A) e^{i\omega t}$$

$$T(A, t) = \bar{T}(A) e^{i\omega t} \rightarrow T(0, t) = b e^{i\omega t}$$

$$\begin{cases} R_0 \frac{\partial^2 U}{\partial t^2} = \frac{\partial T}{\partial A} \\ T + Z_0 \frac{\partial T}{\partial t} = E \left(\frac{\partial U}{\partial A} + Z_1 \frac{\partial^2 U}{\partial A \partial t} \right) \end{cases}$$

$$\Rightarrow \begin{cases} -R_0 \bar{U}(A) \omega^2 = \frac{\partial \bar{T}}{\partial A} \\ \bar{T}(A) = E \frac{1 + i\omega Z_1}{1 + i\omega Z_0} \cdot \frac{d\bar{U}}{dA} \end{cases}$$

$$\begin{cases} \Rightarrow -R_0 (\bar{U}(A)) \omega^2 = \frac{\partial \bar{T}}{\partial A} = E \frac{1 + i\omega Z_1}{1 + i\omega Z_0} \cdot \frac{d^2 \bar{U}}{dA^2} \\ \Rightarrow \frac{d^2 \bar{U}(A)}{dA^2} = -\chi^2 \bar{U} \end{cases}$$

$$\begin{cases} \Rightarrow -R_0 \omega^2 \frac{d\bar{U}(A)}{dA} = \frac{\partial^2 \bar{T}}{\partial A^2} = \bar{T}(A) \frac{1}{E} \cdot \frac{1 + i\omega Z_0}{1 + i\omega Z_1} (-R_0 \omega^2) \\ \Rightarrow \frac{d^2 \bar{T}}{dA^2} = -\chi^2 \bar{T}(A) \quad \bar{T}(A) = \alpha \exp(i\chi A) + \beta \exp(-i\chi A) \end{cases}$$

$$\begin{cases} \bar{T}(A)|_{A=0} = b \\ \lim_{A \rightarrow \infty} \bar{T}(A) = 0 \end{cases} \Rightarrow \bar{T}(A) = b \frac{e^{i\chi A} - e^{-i\chi A + 2i\chi L}}{1 - e^{2i\chi L}} \Big|_{L \rightarrow \infty} = b e^{-i\chi A}$$

$$\text{So. } \bar{U}(A) \Big|_{A=0} = -\frac{i\chi}{R_0\omega^2} b e^{-i\chi A} = \frac{ib\chi}{R_0\omega^2} e^{-i\chi A} \Big|_{A=0} \\ = \frac{ib\chi}{R_0\omega^2}$$

$$U(A=0, t) = \frac{ib\chi e^{i\omega t}}{R_0\omega^2} \\ = \frac{ib e^{i\omega t}}{R_0\omega^2} \cdot \sqrt{\frac{R_0\omega^2}{E} \frac{1+i\omega z_0}{1+i\omega z_1}} \\ = \frac{ib e^{i\omega t}}{\omega} \sqrt{\frac{1}{RE} \frac{1+\omega^2 z_0^2 + i\omega(z_0 - z_1)}{1+\omega^2 z_1^2}} \\ = ib e^{i\omega t} \frac{1}{\omega \sqrt{RE}} \left(\frac{1+\omega^2 z_0^2}{1+\omega^2 z_1^2} \right)^{\frac{1}{4}} e^{i\delta}$$

$$\tan 2\delta = \frac{\omega(z_0 - z_1)}{1+\omega^2 z_0 z_1}$$

$$\text{So. } \begin{cases} U(A=0, t) = b \frac{1}{\omega \sqrt{RE}} \left(\frac{1+\omega^2 z_0^2}{1+\omega^2 z_1^2} \right)^{\frac{1}{4}} \sin(\omega t + \delta - \frac{\pi}{2}) \\ T(A=0, t) = b \sin(\omega t) \end{cases}$$

case imposed displacement: $\tan 2\tilde{\delta} = \frac{\omega(z_1 - z_0)}{1+\omega^2 z_0 z_1}$

eg: $\tilde{\delta} = -\delta$.