Let $\bar{\boldsymbol{\lambda}} = (1 - \boldsymbol{\lambda} \underline{1}_d^T, \boldsymbol{\lambda})$, be a weight vector

Then $\sigma^2_{\bar{\boldsymbol{\lambda}}} = \boldsymbol{\lambda} C \boldsymbol{\lambda}^T$.

Thus we consider the minimization problem

$$\min \quad \boldsymbol{\lambda} C \boldsymbol{\lambda}^T$$

$$\text{s.t.} \quad \boldsymbol{\lambda} \mu^T + (1 - \boldsymbol{\lambda} \underline{1}_d^T) r = \mu_0$$

Consider the Lagrangian

$$L(\boldsymbol{\lambda}, \lambda) = \boldsymbol{\lambda} C \boldsymbol{\lambda}^T + \lambda (\boldsymbol{\lambda} \mu^T + (1 - \boldsymbol{\lambda} \underline{1}_d^T) r - \mu_0).$$

$$= \boldsymbol{\lambda} C \boldsymbol{\lambda}^T + \lambda (\boldsymbol{\lambda} (\mu^T - \underline{1}_d^T r) + r - \mu_0)$$

The first order conditions for the minimum is

$$\frac{\partial L(\boldsymbol{\lambda}, \lambda)}{\partial \boldsymbol{\lambda}} = 2 C \boldsymbol{\lambda}^T + \lambda (\mu^T - \underline{1}_d^T r) = 0 \qquad (1)$$

$$\frac{\partial L(\boldsymbol{\lambda}, \lambda)}{\partial \lambda} = \boldsymbol{\lambda} (\mu^T - \underline{1}_d^T r) + r - \mu_0 = 0 \qquad (2)$$

By (1), we have

$$\lambda^T = -\frac{1}{2}\lambda C^{-1}(\mu^T - \underline{1}_d^T r) \qquad (3)$$

(2) implies

$$\lambda(\mu^T - \underline{1}_d^T r) = (\mu - \underline{1}_d r)\lambda^T = \mu_0 - r.$$

Thus

$$\mu_0 - r = -\frac{1}{2}\lambda(\mu - \underline{1}_d r) C^{-1}(\mu^T - \underline{1}_d^T r)$$

$$\lambda = \frac{-2(\mu_0 - r)}{(\mu - \underline{1}_d r) C^{-1}(\mu^T - \underline{1}_d^T r)}$$

Substitute this into (2), we have

$$\lambda^T = \frac{(\mu_0 - r) C^{-1}(\mu^T - \underline{1}_d^T r)}{(\mu - \underline{1}_d r) C^{-1}(\mu^T - \underline{1}_d^T r)}$$