Q1. (a) Field 1 Refinery 3 customer 5

Field 2 Refinery 4 customer 6

Let amount of oil going through route (1,3,5) be a, (1,3,6) be b, (1,4,5) be c, (1,4,6) be d, (2,3,5) be e, (2,3,6) be f, (2,4,5) be g, (2,4,6) be h.

Then the objective function:

$$\text{Min } 5a + 6b + 6c + 5d + 6e + 7f + 7g + 6h$$

Note that the parameter of each variable is the ~~total~~ shipping cost for each route.

Constraints:

$$a+b+c+d = 5000$$
$$e+f+g+h = 4000$$
$$a+c+e+g = 6000$$
$$b+d+f+h = 3000$$

Note that the left-side equations mean the ~~total~~ amount of oil out of / in each field / customer.

(b) There're new constraints compared to (a). The objective function & decision variables remain rechanged

Add: $\begin{cases} a+b+e+f \leq 6000 \\ c+d+g+h \leq 3500 \end{cases}$

Note that the left-side equations mean the total amount of oil sent to Refinery 3 and 4 respectively.

Q2 (a) Because on all accounts, producing M_GC would be better than M_RC (the final value of M_RC is 0). Decreasing coefficient of M_RC would not change the basis.

(b) 200 > 133.3 (Allowable Decrease). Thus the decrease in profit will affect optimal values of decision variables.

① T_RC ↑ reaches max of 900.

② T_GC ↑ rises to 100.

③ M_GC reaches 800 (stay the same)

Total optimal profit decreases to $(900 \times 940 + 100 \times 1220 + 800 \times 560)$

(c) 150 < 200 (Allowable Increase)

They will not change.

(d) Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$5	Demand Regular Computer (RC)	800	0	900	1E+30	100
\$C\$5	Demand Gaming Computer (GC)	800	0	900	1E+30	100
\$B\$9	Regular Labor Capacity	4000	313.3333333	4000	300	2400
\$B\$10	Specialized Labor Capacity	2400	0	3000	1E+30	600
\$D\$3	Machine capacity Montreal	800	133.3333333	800	100	150
\$D\$4	Machine capacity Toronto	800	—	1000	1E+30	200

- **The final value** is the value of the left-hand side when plugging in the final values for the variables,800 as indicated in the first table;
- **The shadow price** is the dual variable associated with the constraint;
- **The constraint R.H.** side is the right hand side of the constraint, 1000 hours of machine capacity in Toronto;
- **The allowable increase** is by how much the right-hand side of the constraint can increase without changing the basis, 1E+30(infinity);
- **The allowable decrease** is by how much the right-hand side of the constraint can decrease without changing the basis, 200, which is calculated by 1000-800 = 200.

Q3. (a) Let quantity of beer 1 be x, beer 2 be y, beer 3 be z. (litre)

Then objective function (to maximize profit)

$$\text{Max } 2.5x + 3.7y + 2.8z$$

given

$0.4x + 0.8y + 0.4z \le 4000$ (Brew stage constraint)

$0.4x + 0.6y + 0.5z \le 3800$ (Bottling stage constraint)

$$\left.\begin{array}{l} x \le 2000 \\ y \le 3400 \\ z \le 1800 \end{array}\right\} \text{(max demand constraint)}$$

(b) Yes, based on processing time, we can get the hourly production profit for each type of beer

beer 1 : $2.5/(0.4+0.4) = \$3.125/hr$

beer 2 : $3.7/(0.8+0.6) = \$2.64/hr$

beer 3 : $2.8/(0.4+0.5) = \$3.11/hr$

Thus, we should produce beer 1 till maximum, then beer 3, then beer 2

Thus, the optimal decisions: 2000 litres beer 1, 1800 litres beer 3, and beer 2 with 3100 litres

$$\min \left\{ \frac{4000-800-720}{0.8}, \frac{3800-800-900}{0.6}, 3400 \right\}$$

$= 3100$

3.2 (a) New objective function:

$$\text{Max } 2.5x + 3.7y + 2.8z - (\underbrace{4000 \times 1_x + 3000 \times 1_y + 2000 \times 1_z}_{\text{Set-up costs}})$$

where $1_x = \begin{cases} 1, & \text{when } x > 0 \\ 0, & \text{when } x = 0 \end{cases}$

All the other conditions remain the same.

(b) compared to 3.2.a. the only change is in the brewing stage constraint:

$$0.4x + 0.8y + 0.4z + \underbrace{100\,1_x + 50\,1_y + 75\,1_z}_{\text{Set-up time}} \le 4000$$

where $1_x = \begin{cases} 1, & \text{when } x > 0 \\ 0, & \text{when } x = 0 \end{cases}$

Q4. (a) Let W_{LH} denote the weight attached to Labor Hours
W_{OC} denote the weight attached to Operating Cost
W_{NL} denote the weight attached to New Loans
W_{sat} denote the weight attached to Satisfaction

objective function:

$$\text{Max } (770 W_{NL} + 97 W_{sat})/(373 W_{LH} + 5340 W_{OC})$$

subject to $330 W_{LH} + I340 W_{OC} = 1$

$$0 \leq (770 W_{NL} + 97 W_{sat})/(373 W_{LH} + 5340 W_{OC}) \leq 1$$

$$0 \leq (780 W_{NL} + 94 W_{sat})/(349 W_{LH} + 4430 W_{OC}) \leq 1$$

$$0 \leq (790 W_{NL} + 93 W_{sat})/(598 W_{LH} + 5310 W_{OC}) \leq 1$$

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(b) No. because DEA analysis would select the best weight that maximize efficiency for each branch.

(c) Based on the given info on inputs and outputs, compared with unit 6, unit 9 yields lower new loans and higher satisfaction based on higher labor hours and lower operating costs. They're both on the efficient frontier with differentiated capabilities.

(d) No. Because with all conditions staying the same, unit 3 cannot get improvement from input cut. I would suggest learn from Unit 9.