1.

a. 令 $u=2x+6$，则 $x=\frac{1}{2}(u-6)$

$\int(2x+6)^5dx$

$=\int u^5 d(\frac{1}{2}u-6)=\frac{1}{2}\int u^5du$

$=\frac{1}{2}\cdot\frac{1}{6}u^6+C=\frac{1}{12}u^6+C$

$=\frac{1}{12}(2x+6)^6+C$

b. 令 $u=3x+5$，则 $x=\frac{1}{3}(u-5)$

$\int\frac{1}{3x+5}dx$

$=\int\frac{1}{u}d[\frac{1}{3}(u-5)]=\frac{1}{3}\int\frac{1}{u}du$

$=\frac{1}{3}\ln u+C=\frac{1}{3}\ln(3x+5)+C$

c. $\int(3x^2-1)e^{x^3-x}dx$

$=\int \cancel{3x^2} e^{x^3-x}d(x^3-x)$

令 $u=x^3-x$ ①

① $=\int e^u du=e^u+C$

$=e^{x^3-x}+C$

d. $\int\frac{10x^3-5x}{\sqrt{x^4-x^2+6}}dx$

$=\frac{10}{4}\int\frac{4x^3-2x}{\sqrt{x^4-x^2+6}}dx$

$=\frac{10}{4}\int\frac{1}{\sqrt{x^4-x^2+6}}d(x^4-x^2+6)$ ①

令 $u=x^4-x^2+6$.

① $=\frac{10}{4}\int\frac{1}{\sqrt{u}}du=\frac{10}{4}\cdot 2u^{\frac{1}{2}}+C$

$=5(x^4-x^2+6)^{\frac{1}{2}}+C$

e. $\int\frac{2x\ln(x^2+1)}{x^2+1}dx$

$=\int\frac{\ln(x^2+1)}{x^2+1}d(x^2+1)$ ①

令 $u=x^2+1$

① $=\int\frac{\ln u}{u}du=\frac{1}{2}(\ln u)^2+C$

$=\frac{1}{2}(\ln(x^2+1))^2+C$

f. $\int\frac{x}{2x+1}dx$

$=\frac{1}{2}\int\frac{1}{2x+1}d(2x+1)$ ①

令 $u=2x+1$

① $=\frac{1}{2}\int\frac{1}{u}du=\frac{1}{2}\ln u+C$

$=\frac{1}{2}\ln(2x+1)+C$

2.

(a) $x(t)=\int x'(t)dt=-\int\frac{2t}{(1+t^2)^{\frac{3}{2}}}dt$

$=-\int\frac{1}{(1+t^2)^{\frac{3}{2}}}d(t^2+1)$ ①

令 $u=1+t^2$

① $=\cancel{-\int\frac{1}{u^{\frac{3}{2}}}}\ -\int\frac{1}{u^{\frac{3}{2}}}du=\frac{1}{2}u^{-\frac{1}{2}}+C$

$=\frac{1}{2}(1+t^2)^{-\frac{1}{2}}+C$

已知，$x(0)=4$，即

$x(0)=\frac{1}{2}(1+0^2)^{-\frac{1}{2}}+C=4$

即 $C=\frac{7}{2}$

即 $x(t)=\frac{1}{2}(1+t^2)^{-\frac{1}{2}}+\frac{7}{2}$

(b) $x(t) = \frac{1}{2}(1+t^2)^{-\frac{1}{2}} + \frac{7}{2}$

则 $x(4) = \frac{1}{2}(1+4^2)^{-\frac{1}{2}} + \frac{7}{2}$

$= \frac{1}{2\sqrt{17}} + \frac{7}{2}$

3(a) $R(x) = \int R'(x)\,dx$

$= \int 50 + 3.5xe^{-0.01x^2}\,dx$

$= 50x + 3.5\int xe^{-0.01x^2}\,dx$

$= 50x + 3.5\times\frac{1}{2}\int (e^{-0.01})^{x^2}\,dx^2$ ①

设 $u = x^2$. 则

① $= 50x + 3.5\times\frac{1}{2}\int (e^{-0.01})^{u}\,du$

$= 50x + 3.5\times\frac{1}{2}e^{-0.01u} + C$

$= 50x + \frac{7}{4}e^{-0.01x^2} + C$

又: $R(0) = 0$.

即 $R(0) = 50\times 0 + \frac{7}{4}e^{-0.01\times 0^2} + C =$

即 $C = -\frac{7}{4}$

(b) $R(x) = 50x + \frac{7}{4}e^{-0.01x^2} - \frac{7}{4}$

$R(1000) = 50\times 1000 + \frac{7}{4}e^{-0.01\times 1000^2} - \frac{7}{4}$

$= 2428.32$

4. (a) $\int_0^1 e^{-x}(4-e^x)\,dx$

$= 4\int_0^1 e^{-x}\,dx - \int_0^1 1\,dx$

$= 4\cdot e^{-x}\Big|_0^1 - x\Big|_0^1$

$= \frac{4}{e} - 5$

(b) $\int_1^3 (1+\frac{1}{x}+\frac{1}{x^2})\,dx$

$= \int_1^3 1\,dx + \int_1^3 \frac{1}{x}\,dx + \int_1^3 \frac{1}{x^2}\,dx$

$= x\Big|_1^3 + \ln x\Big|_1^3 + \ln x^2\Big|_1^3$

$= 2 - \ln 3 - \ln 9$.

(c) $\int_0^1 (x^3+x)\sqrt{x^4+2x^2+1}\,dx$

$= \frac{1}{4}\int_0^1 \sqrt{x^4+2x^2+1}\,d(x^4+2x^2+1)$ ①

设 $u = x^4+2x^2+1$

① $= \frac{1}{4}\int_1^4 \sqrt{u}\,du$

$= \frac{1}{4}\times\frac{2}{3}u^{\frac{3}{2}}\Big|_1^4$

$= \frac{1}{6}(4^{\frac{3}{2}} - 4)$

(d) $\int_{-3}^{-1} \frac{t+1}{t^3}\,dt$

$= \int_{-3}^{-1} \frac{1}{t^2}\,dt + \int_{-3}^{-1} \frac{1}{t^3}\,dt$

$= \ln t^2\Big|_{-3}^{-1} + \ln t^3\Big|_{-3}^{-1}$

$= \ln 1 - \ln 9 + \ln(-1) - \ln(-27)$

5. (a) $\int_{-3}^{1}[4f(x)-3g(x)]\,dx$

$=4\int_{-3}^{1}f(x)-3\int_{-3}^{1}g(x)\,dx$

$=4\times 0-3\times 4=-12$

(b) $\int_{4}^{4}g(x)\,dx$

$=0$

(c) $\int_{2}^{-3}f(x)\,dx$

$=-\int_{-3}^{2}f(x)\,dx=-5$

6. (a) $\int_{\frac{1}{3}}^{\frac{1}{2}}\frac{e^{\frac{1}{x}}}{x^2}\,dx$

$=-\int_{\frac{1}{3}}^{\frac{1}{2}}e^{\frac{1}{x}}\,d\left(\frac{1}{x}\right)$ ①

$=-e^{\frac{1}{x}}\Big|_{\frac{1}{3}}^{\frac{1}{2}}=-e^{2}+e^{3}$

$=e^{3}-e^{2}$

(b) $\int_{1}^{e^2}Q'(x)\,dx=\int_{1}^{e^2}\frac{(\ln x)^2}{x}\,dx$

$=\int_{1}^{e^2}(\ln x)^2\,d(\ln x)$

$=\frac{1}{3}(\ln x)^3\Big|_{1}^{e^2}$

$=\frac{1}{3}\times(\ln e^2)^3-\frac{1}{3}\times(\ln 1)^3$

$=\frac{1}{3}\times 8=\frac{8}{3}$

7. $\int_{4}^{6}V'(t)\,dt=\int_{4}^{6}12e^{-0.05t}(e^{0.3t}-3)\,dt$

$=12\int_{4}^{6}e^{0.25t}\,dt-36\int_{4}^{6}e^{-0.05t}\,dt$

$=48\int_{4}^{6}e^{0.25t}\,d(0.25t)+720\int_{4}^{6}e^{-0.05t}\,d(-0.05t)$

$=48\times e^{0.25t}\Big|_{4}^{6}+720\times e^{-0.05t}\Big|_{4}^{6}$

$=28.55$ (thousands of dollars)