1. a. $F(y) = (\frac{1}{y^2} - \frac{3}{y^4})(y+5y^3)$

$dF(y) = F'(y)\,dy$

$= [(\frac{1}{y^2} - \frac{3}{y^4})'(y+5y^3) + (\frac{1}{y^2} - \frac{3}{y^4})(y+5y^3)'\,]\,dy$

$= [(\frac{-2}{y^3} - \frac{-12}{y^5})(y+5y^3) + (\frac{1}{y^2} - \frac{3}{y^4})(1+15y^2)\,]\,dy$

$= [\frac{-2}{y^2} + \frac{12}{y^4} + (-10) + \frac{60}{y^2} + \frac{1}{y^2} - \frac{3}{y^4} + 15 - \frac{45}{y^2}\,]\,dy$

$= (\frac{9}{y^4} + \frac{14}{y^2} + 5\,)\,dy$

b. $f(x) = \frac{x^2 e^x}{\ln x + e^x}$

$df(x) = f'(x)\,dx$

$= \frac{(x^2e^x)'(\ln x + e^x) - (x^2e^x)(\ln x + e^x)'}{(\ln x + e^x)^2}\,dx$

$= \frac{(2x\cdot e^x + x^2e^x)(\ln x + e^x) - (x^2e^x)(\frac{1}{x} + e^x)}{(\ln x + e^x)^2}\,dx$

$= \frac{2xe^x\ln x + x^2e^x\ln x + 2xe^{2x} + x^2e^{2x} - xe^x - x^2e^{2x}}{(\ln x)^2 + e^{2x} + 2\ln x\cdot e^x}\,dx$

$$= \frac{(x^2+2x)e^x \ln x + 2x e^{2x} - x e^x}{(\ln x)^2 + e^{2x} + 2e^x \ln x} \, dx$$

c. $f(x) = \dfrac{1-5x^2}{\sqrt[3]{3+2x}}$

$$df(x) = f'(x)\,dx$$

$$= \frac{(1-5x^2)'(3+2x)^{\frac{1}{3}} - (1-5x^2)\left[(3+2x)^{\frac{1}{3}}\right]'}{(3+2x)^{\frac{2}{3}}} \, dx$$

$$= \frac{(\cancel{1}-10x)(3+2x)^{\frac{1}{3}} - (1-5x^2)\cdot\frac{1}{3}(3+2x)^{-\frac{2}{3}}\cdot 2}{(3+2x)^{\frac{2}{3}}} \, dx$$

$$= \frac{-10x\cdot\sqrt[3]{3+2x} - \frac{2}{3}(1-5x^2)(3+2x)^{-\frac{2}{3}}}{\sqrt[3]{(3+2x)^2}} \, dx$$

$$d.\quad y = 3^{[e^{(x^2)}]}$$

$$dy = y'\,dx$$

$$= 3^{e^{x^2}} \ln 3 \cdot (e^{x^2})'\,dx$$

$$= 3^{e^{x^2}} \ln 3 \cdot e^{x^2} \cdot \ln e \cdot (x^2)'\,dx$$

$$= 3^{e^{x^2}} \ln 3 \cdot e^{x^2} \cdot 2x\,dx$$

$$= \underline{2\ln 3} \cdot x e^{x^2} \cdot 3^{e^{x^2}}\,dx$$

2.(a) 利润 = 总售价 − 总成本价

= 单位售价 × 销量 − 总成本价.

$$\therefore P(p) = p \cdot \frac{500}{p+3} - (0.2p^2 + 3p + 200)$$

$$= \frac{500p}{p+3} - 0.2p^2 - 3p - 200.$$

(b)
$$P'(p) = \frac{(500p)'(p+3) - 500p(p+3)'}{(p+3)^2} - 0.4p - 3$$

$$= \frac{500(p+3) - 500p}{(p+3)^2} - 0.4p - 3$$

$$= \frac{1500}{(p+3)^2} \underline{- 0.4p - 3}$$

当 $p = 12$ 时，代入上式.

$$P'(12) = \frac{1500}{(12+3)^2} - 0.4 \times 12 - 3 = \cancel{100}\ \underline{\frac{100}{15} - 4.8 - 3}$$

$$= \cancel{3.1867}\ \underline{-1.33}$$

$\because P'(12) = -1.33 < 0$

$\therefore$ 利润在下降.

3. (a) $A = 10000\left(1+\frac{r}{52}\right)^{520}$

$A' = 10000 \times \underline{520 \times \left(1+\frac{r}{52}\right)^{520-1}} \times \frac{1}{52}$

$= 10000 \times 520\left(1+\frac{r}{52}\right)^{519} \cdot \frac{1}{52}$

$= 100000\left(1+\frac{r}{52}\right)^{519}$

(b) 当 $r = \underline{0.05}$ 时，代入上式 A' 得.

$A' = 100000 \times \left(1+\frac{0.05}{52}\right)^{519}$

$= 164674.18$

4. $Q = 3000K^{\frac{1}{2}}L^{\frac{1}{3}}$, ~~K = 400000 , L = 1331~~

$$dQ = 3000\left(\frac{1}{2}K^{-\frac{1}{2}}\cdot L^{\frac{1}{3}}\,dK + K^{\frac{1}{2}}\cdot\frac{1}{3}L^{-\frac{2}{3}}\,dL\right)$$

$$= 1500K^{-\frac{1}{2}}L^{\frac{1}{3}}\,dK + 1000K^{\frac{1}{2}}L^{-\frac{2}{3}}\,dL \qquad ①$$

据题意，增加额外\$1000投资，即 $dK = 1$.

人力资源不变，即 $dL = 0$.

当前投资为 400000，即 $K = 400$.

工时为 1331，即 $L = 1331$. 代入①式.

$$dQ = 1500\times 400^{-\frac{1}{2}}\times 1331^{\frac{1}{3}}\times 1 + 1000\times 400^{\frac{1}{2}}\times 1331^{-\frac{2}{3}}\times 0$$

$$= \frac{1500\times\sqrt[3]{1331}}{\sqrt{400}} = \frac{1500\times 11}{20} = 825.$$

∴ 每日产量变化为 825 单位.

5. $C(q) = \frac{1}{6}q^3 + 642q + 400$, $dC = -130$, $q = 4$.

$$dC = C'\,dq = \left(\frac{1}{6}\times 3q^2 + 642\right)dq = \left(\frac{1}{2}q^2 + 642\right)dq.$$

$$-130 = \left(\frac{1}{2}q^2 + 642\right)dq$$

$\cancel{\frac{1}{2}q^2 =}$ $-130 = \left(\frac{1}{2}\times 4^2 + 642\right)dq.$

$$-130 = \cancel{6} \, 650\,dq$$

$$dq = -0.2 \qquad\qquad 4 - 0.2 = 3.8$$

∴ 生产商应减少 0.2 单位~~生产~~产品的生产，即生产 3.8 单位.

6. $G(x)=x^2-\frac{1}{x^2}$. 定义域$\{x\neq 0$且$x\in R\}$.

$G'(x)=2x+2x^{-3}$

当$G'(x)>0$，即$2x+2x^{-3}>0$

解得 $x>0$

当$G'(x)<0$，即$2x+2x^{-3}<0$

解得$x<0$.

∴ 函数增区间为$(0,+\infty)$，减区间为$(-\infty,0)$

7. $F(x)=\dfrac{x^2}{x-1}$ 　　定义域 $(-\infty,1)\cup(1,+\infty)$

$$F'(x)=\frac{(x^2)'(x-1)-x^2(x-1)'}{(x-1)^2}$$

$$=\frac{2x(x-1)-x^2}{x^2-2x+1}$$

$$=\frac{2x^2-2x-x^2}{x^2-2x+1}$$

$$=\frac{x^2-2x}{x^2-2x+1}$$

令 $F'(x)=0$，即 $\dfrac{x^2-2x}{x^2-2x+1}=0$. 解得 $x_1=0$，$x_2=2$.

当 $x<0$ 时，$F'(x)>0$

当 $0<x<2$ 且 $x\neq1$ 时. $F'(x)<0$.

当 $x>2$ 时. $F'(x)>0$.

∴ 当 $x=0$ 时. 函数有极大值 $F(0)=0$.

当 $x=2$ 时. 函数有极小值 $F(2)=4$.

	$(-\infty,0)$	$[0,1)$	$(1,2]$	$(2,+\infty)$
$F(x)$	↗	↘	↘	↗