Problem 2.1

$\sqrt[3]{x+y} + xy^2 = 2x + y$ 即 $(x+y)^{\frac{1}{3}} + xy^2 = 2x + y$.

同时对方程两边求导.

$\frac{1}{3}(x+y)^{-\frac{2}{3}} \cdot (1+y') + y^2 + x \cdot 2y \cdot y' = 2 + y'$

$\frac{1}{3}(x+y)^{-\frac{2}{3}}(1+y') + y^2 + 2xyy' = 2 + y'$

将 $x=2$, $y=-1$ 代入上式得

$\frac{1}{3}(2-1)^{-\frac{2}{3}}(1+y') + (-1)^2 + 2\times2\times(-1)y' = 2 + y'$

$\frac{1}{3}(1+y') + 1 - 4y' = 2 + y'$

$\frac{1}{3} + \frac{1}{3}y' + 1 - 4y' = 2 + y'$

$\cancel{\frac{2}{3} = \frac{14}{3}}$ $\frac{1}{3} + 1 - 2 = y' + 4y' - \frac{1}{3}y'$

$-\frac{2}{3} = \frac{14}{3}y'$

$y' = -\frac{1}{7}$

切线 $y = mx + b$ 即 $y = -\frac{1}{7}x + b$.

将 $(x, y) = (2, -1)$ 代入上式得

$-1 = -\frac{1}{7} \times 2 + b$ 解得 $b = -\frac{5}{7}$

$\therefore y = -\frac{1}{7}x - \frac{5}{7}$

Problem 2.2

$$f(x)=\begin{cases} x(\sin x)(\sin\frac{1}{x}) & , x\neq 0 \\ 0 & , x=0 \end{cases}$$

(a) 当 $x\neq 0$ 时，$f(x) = x(\sin x)(\sin\frac{1}{x})$

$$f'(x) = [(x\sin x)\sin\frac{1}{x}]'$$

$$= (x\sin x)'\sin\frac{1}{x} + (\sin\frac{1}{x})'\cdot(x\sin x)$$

$$= [x'\sin x + x(\sin x)']\sin\frac{1}{x} + (\sin\frac{1}{x})'\cdot x\sin x$$

$$= (\sin x + x\cos x)\sin\frac{1}{x} + (\cos\frac{1}{x})\cdot(-\frac{1}{x^2})\cdot x\sin x.$$

$$= \sin x\cdot\sin\frac{1}{x} + x\cos x\cdot\sin\frac{1}{x} - \cos\frac{1}{x}\cdot\frac{1}{x^2}\cdot x\sin x$$

(b) $\because f'(x_0) = \lim\limits_{\Delta x\to 0}\dfrac{f(x_0+\Delta x)-f(x_0)}{\Delta x}.$

$$\therefore f'(0) = \lim_{\Delta x\to 0}\frac{f(\Delta x)-f(0)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta x(\sin\Delta x)(\sin\frac{1}{\Delta x})-0}{\Delta x}$$

$$= \lim_{\Delta x\to 0}(\sin\Delta x)(\sin\frac{1}{\Delta x}) = 0.$$

$\therefore f(x)$ 在 $x=0$ 处可微.（一元函数可导必可微）

Problem 2.3

$$F = \frac{GMm}{r^2}.$$

$$\frac{dF}{dt} = \frac{(GMm)'r^2 - (GMm)(r^2)'}{(r^2)^2}$$

$$= \frac{GM\frac{dm}{dt}r^2 - GMm \cdot 2r \cdot \frac{dr}{dt}}{r^4}$$

$$= \frac{GM\frac{dm}{dt}r - 2GMm \cdot \frac{dr}{dt}}{r^3}$$