European Options:

$$maximum\ call = c_t \leq S_t, \qquad minimum\ call = c_t \geq \max\left(0, S - \frac{X}{\left(1+r_f\right)^{T-t}}\right)$$

$$maximum\ put = p_t \leq \frac{X}{\left(1+r_f\right)^{T-t}}, \qquad minimum\ put = p_t \geq \max\left(0, \frac{X}{\left(1+r_f\right)^{T-t}} - S\right)$$

American Options:

$$maximum\ call = C_t \leq S_t, minimum\ call = C_t \geq \max\left(0, S - \frac{X}{\left(1+r_f\right)^{T-t}}\right)$$

$$maximum\ put = P_t \leq X, \qquad minimum\ put = \max(0, X - S)$$

If $C < c$ or $P < p$ at the time $t < T$, then American Option holder could wait until its maturity and hence $c = C, p < P$ since both options exercise at the same time.
If $X > X_T$ for put and $X < X_T$ for call, then American call and put writer can exercise their options before maturity and hence $c < C, p < P$.
Therefore, $c \leq C$ and $p \leq P$