

$$a) \bar{\gamma} = \sum_{i=1}^4 \gamma_i P(\gamma_i) \\ = 15 \text{ dB}$$

$$C_{\text{AWGN}} = B \log_2 (1 + \bar{\gamma}) = 55.4518 \text{ Mbps}$$

$$b) C_1 = \sum_i B \log_2 (1 + \gamma_i) P(\gamma_i) \\ = 46.3905 \text{ Mbps}$$

$$c) C_0 = (1 - P_{\text{out}}) B \log_2 (1 + \gamma_{\min}) \quad , \quad P_{\text{out}} = P(\gamma < \gamma_{\min})$$

$\gamma_{\min} > 30 \text{ dB}$	$P_{\text{out}} = 1$	$C_0 = 0$
$20 \text{ dB} < \gamma_{\min} \leq 30 \text{ dB}$	$P_{\text{out}} = 0.8$	$C_{0-\text{max}} = C_0 (\gamma_{\min} = 30 \text{ dB}) = 13.7359 \text{ Mbps}$
$10 \text{ dB} < \gamma_{\min} \leq 20 \text{ dB}$	$P_{\text{out}} = 0.5$	$C_{0-\text{max}} = C_0 (\gamma_{\min} = 20 \text{ dB}) = 30.4432 \text{ Mbps}$
$0 < \gamma_{\min} \leq 10 \text{ dB}$	$P_{\text{out}} = 0.2$	$C_{0-\text{max}} = C_0 (\gamma_{\min} = 10 \text{ dB}) = 38.3663 \text{ Mbps}$
$\gamma_{\min} = 0$	$P_{\text{out}} = 0$	$C_0 = 0$

Consider water-filling

$$\max C = \sum_i B \log_2 \left(1 + \frac{\gamma_i p_i}{\bar{P}} \right) P(\gamma_i)$$

$$\text{s.t.} \quad \sum_i p_i P(\gamma_i) = \bar{P}$$

$$\text{so.} \quad \frac{p_i}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} & \gamma_i \geq \gamma_0 \\ 0 & \gamma_i < \gamma_0 \end{cases}$$

$$\gamma_0 > 0. \quad \frac{0.8}{\gamma_0} = 1 + \sum_{i=1}^3 \frac{P(\gamma_i)}{\gamma_i}$$

$$\gamma_0 = 0.7607 > \gamma_4 = 0$$

$$\text{so. } C = \sum_{i=1}^5 B \log_2 \left(\frac{\gamma_i}{\gamma_0} \right) p(\gamma_i) \\ = 50.5350 \text{ Mbps}$$

cd). so. $C_{\text{water-filling}} > C_1$, both transmitter and receiver
 $C_{\text{water-filling}} < C_{\text{MIMO}}$. have CSI, we can use
 water-filling method. the performance of the communication
 system will be improved. but it can't reach C_{MIMO} .