1. $a_n = \left[\left(1+\frac{1}{n}\right)^n \right]^{n+1}$

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} e^{n+1} = \infty$$

2. $b_n = \dfrac{n^3 + 7^n + n!\cos(3^{n/2}\pi)}{\pi\log n + n + 2^n + n!}$

$$= \frac{\frac{n^3}{n!} + \frac{7^n}{n!} + \cos(3^{n/2}\pi)}{\frac{\pi\log n}{n!} + \frac{n}{n!} + \frac{2^n}{n!} + 1}$$

$n! \approx \left(\frac{n}{e}\right)^n$ For $n \gg 1$ Stirling's approximation

$$\lim_{n\to\infty} \frac{n^3}{n!} = 0 \qquad \lim_{n\to\infty} \frac{7^n}{n!} = 0$$

$$\lim_{n\to\infty} \frac{\pi\log n}{n!} = 0 \qquad \lim_{n\to\infty} \frac{n}{n!} = 0 \qquad \lim_{n\to\infty} \frac{2^n}{n!} = 0$$

$$\lim_{n\to\infty} b_n = \frac{0+0+\lim_{n\to\infty} \cos(3^{n/2}\pi)}{0+0+0+1}$$

oscilate between
-1 and 1

$= \text{indeterminate}$

3. $$c_n = \frac{n\sin(2^n)}{n^2+1}$$

$$= \frac{\sin(2^n)}{n+\frac{1}{n}}$$

$$-1 \le \sin(2^n) \le 1$$

$$\lim_{n\to\infty} n+\frac{1}{n} \to \infty$$

$$\frac{-1 \le \sin(2^n) \le 1}{\infty} = 0$$

therefore $\lim_{n\to\infty} c_n = 0$