1、 $\pi_2 = (p_2-5)(21+2p_1-3p_2) = 36p_2+2p_1p_2-3p_2^2-10p_1-105$

to make π_2 max:

$\frac{\partial \pi_2}{\partial p_2} = 36+2p_1-6p_2=0$

$p_2=\frac{1}{3}p_1+6$

$p_1=8$

$p_2=\frac{26}{3}\approx 8.67$

$\therefore$ the closest integer to the optimal price level multiplied by 100 is 867.

2、 $\pi_1=(p_1-5)(21-3p_1+2p_2)=36p_1-10p_2+2p_1p_2-3p_1^2-105$

to make π_1 max:

$\frac{\partial \pi_1}{\partial p_1}=36+2p_2-6p_1=0$

$p_1=\frac{1}{3}p_2+6$

$\pi_2=(p_2-5)(21+2p_1-3p_2)=36p_2+2p_1p_2-3p_2^2-10p_1-105$

$\pi_2^N=\frac{\partial \pi_2}{\partial p_2}=36+2p_1-6p_2=0$

$p_2=\frac{1}{3}p_1+6$

$\begin{cases} p_1=\frac{1}{3}p_2+6 \\ p_2=\frac{1}{3}p_1+6 \end{cases} \Rightarrow \begin{cases} p_1=9 \\ p_2=9 \end{cases}$

$100\times\pi_2^N=9\times 100=900$

the closest integer to $100\times\pi_2^N$ is 900.

3、 when $d=I$:

$\pi_1=(p_1-1)(21-3p_1+2p_2)-50=24p_1-3p_1^2+2p_1p_2-2p_2-71$

to make π_1 max:

$\frac{\partial \pi_1}{\partial p_1}=24-6p_1+2p_2=0$

$p_1=\frac{1}{3}p_2+4$

$\pi_2=36p_2+2p_1p_2-3p_2^2-10p_1-105$

to make π_2 max:

$\frac{\partial \pi_2}{\partial p_2}=36+2p_1-6p_2=0$

$p_2=\frac{1}{3}p_1+6$

$\begin{cases} p_1=\frac{1}{3}p_2+4 \\ p_2=\frac{1}{3}p_1+6 \end{cases} \Rightarrow \begin{cases} p_1=6.75 \\ p_2=8.25 \end{cases}$

$\pi_1=(p_1-1)(21-3p_1+2p_2)=49.875$

$\pi_2=(p_2-5)(21-3p_2+2p_1)=31.6875$

when $d = N$:

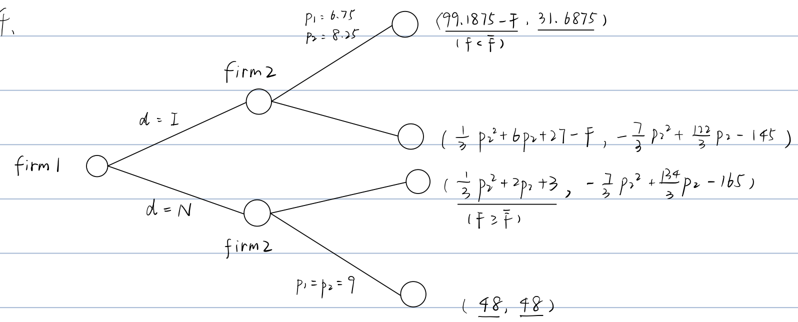
$p_1 = p_2 = 9$

$\pi_1 = \pi_2 = 48$

$\therefore \pi_1^{*} = \pi_1^{2} = 49.1875$

the closest integer to $100 \times \pi_1^{*}$ is 49.9.

4.



when $d = I$:

to make π_1 max, $p_1 = \frac{1}{3}p_2 + 4$

then $\pi_1 = (\frac{1}{3}p_2 + 3)(21 - p_2 - 12 + 2p_2) - \bar{f} = \frac{1}{3}p_2^2 + 6p_2 + 27 - f$

$\pi_2 = -\frac{7}{3}p_2^2 + \frac{122}{3}p_2 - 145$

when $d = N$:

to make π_1 max, $p_1 = \frac{1}{3}p_2 + 6$

then $\pi_1 = (\frac{1}{3}p_2 + 1)(21 - p_2 - 18 + 2p_2) = \frac{1}{3}p_2^2 + 2p_2 + 3$

$\pi_2 = (p_2 - 5)(21 + \frac{2}{3}p_2 + 12 - 3p_2) = -\frac{7}{3}p_2^2 + \frac{134}{3}p_2 - 165$

so we can know :

① firm 2 can change p_2 to make $\pi_1 = \frac{1}{3}p_2^2 + 6p_2 + 27 - f < 48$, firm 1 chooses "$d = N$"

② $p_2 = 8.25$, $\pi_1 = 42.1875$, if and only if $99.1875 - \bar{f} < 42.1875$, firm 1 chooses "$d = N$", there is a Nash equilibrium

statement 3 is true.