1.

$$F_X(x) = P(X \le x)$$

$$= \int_1^x \frac{\lambda}{y^{\lambda+1}}\, dy$$

$$= -y^{-\lambda} \Big|_1^x$$

$$= 1 - x^{-\lambda} \qquad x > 1$$

$$F_Y(y) = P(Y \le y)$$

$$= P(\log X \le y)$$

$$= P(X \le e^y)$$

$$= 1 - e^{-\lambda y}, \qquad y > 0$$

$f_Y(y) = \lambda e^{-\lambda y} \qquad y > 0$

2. The range of $X \mid X+Y$ is $(0, x+y)$,

The joint density of (X, Y) is

$$f_{X,Y}(x, y) = \exp(-x-y) \qquad x > 0,\ y > 0$$

Let $Z = X + Y$, then the joint density of (X, Z) is

$$f_{X,Z}(x, z) = \exp(-z). \qquad z > x > 0.$$

The marginal density of Z is

$$f_Z(z) = \int_0^z \exp(-z)\, dx$$

$$= z\exp(-z) \qquad z > 0.$$

Thus,

$$f_{X|Z}(x) = \frac{f_{X,Z}(x,z)}{f_Z(z)}$$

$$= \frac{\exp(-z)}{z\exp(-z)} \qquad 0 < x < z$$

$$= \frac{1}{z} \qquad 0 < x < z.$$

Thus $X \mid X+Y$ is a uniform distribution on $(0, x+y)$.