Q1.

The risk priority number is calculated by

$$RPN = S \times O \times D.$$

So, the two causes with the highest RPN are "motor fails" and "blocked airflow from dust build up", which have RPN of 36 and 60 respectively.

For the cause "motor fails", the risk can be reduced by reducing the effects of failure. (S)

For the cause "blocked airflow from dust build up", the risk can be reduced by reducing the likelihood of the occurrence of the failure. (O)

Q2.

$$P(\text{cost is greater than £1.2 Billion}) = \int_{1.2}^{\infty} \frac{n x^{n-1}}{(1+x^n)^2}\, dx$$

$$= -\frac{1}{x}\Big|_{1.2}^{\infty}$$

$$= \frac{1}{1.2}$$

$$\approx 0.833.$$

Q3.

a. The 25th precentile of the data is $\frac{93.9+94.3}{2} = 94.1$.

the 50th percentile of the data is $\frac{96.9+98.2}{2} = 97.55$

the 75th percentile of the data is $\frac{98.5+99.3}{2} = 98.9$.

The interquartile range is 4.8.

So, the data point 85.8 is out of the 1.5 IQR below the 25th percentile, and it is an outlier.

b. The sample variance $S_n^2 = \frac{\sum_i (X_i - \bar{X})^2}{n-1} \sim \frac{\sigma^2}{n-1} \chi^2(n-1)$, where σ^2 is the true variance.

So, the 95% 2-sided CI of σ is.

$$\left[\sqrt{\frac{(n-1)S_n^2}{\chi^2_{0.975}(n-1)}}, \sqrt{\frac{(n-1)S_n^2}{\chi^2_{0.025}(n-1)}}\right].$$

Substitute $S_n^2 = 10.48$, $n=7$, $\chi^2_{0.975}(6)=14.45$,

$\chi^2_{0.025}(6) = 1.24$.

the 95% 2-sided CI of σ is

$[2.09, 7.11.]$.

Q4. $P(\text{failure by yielding}) = P(\text{maximum stress} \leq \text{yield strength}) \leq 10^{-5}$.

$\Leftrightarrow$ $\sigma_{max} \leq \alpha_{10^{-5}}$, where $\alpha_{10^{-5}}$ is the 10^{-5} quantile of the lognormal distribution with mean of 337 MPa and CoV of 0.07.

For lognormal distribution with parameter μ and σ^2, its mean is $\exp(\mu + \frac{\sigma^2}{2})$, and variance of $[\exp(\sigma^2)-1]\exp(2\mu+\sigma^2)$.

So, $\sigma^2 = 0.0049$ $\quad \mu = 5.8176$

$\Rightarrow \sigma_{max} \leq \exp(\mu + \sqrt{2\sigma^2}\,\mathrm{erf}^{-1}(2\times10^{-5}-1)) \approx 249.40\text{ MPa}$

$\Rightarrow m_{max} \leq \dfrac{\sigma_{max} A}{g} \approx 20446.42\text{ kg}$.

Q5. Considering the expected cost of failure before and after adding stand-by components of A, B, C,

$E_{original} = C(1-(1-p_A)(1-p_B)(1-p_C))$

≈ 8496

$$E_A = C(1-(1-p_A^2)(1-p_B)(1-p_C))$$

$$\approx 3504$$

$$E_B = C(1-(1-p_A)(1-p_B^2)(1-p_C)) \approx 7498$$

$$E_C = C(1-(1-p_A)(1-p_B)(1-p_C^2)) \approx 6000.$$

So, the option with the highest return is.

$$r_C = E_{original} - E_C - cost_C \approx 1496$$

The component C should have a stand-by.

Q6. Within a period of $\frac{1}{4}$ months.

E(risk associated with significant damage)

$=$[P(sensor detected, relief valve automatic failure, manual detection success, relief valve manual failure)

$+$P(sensor detected, relief valve automatic failure, manual detection failure)

$+$P(sensor failure, manual detection success, relief valve manual failure)

$+$P(sensor failure, manual detection failure)] $\times$ l_{major}

$$= (0.9999 \times 0.005 \times 0.99 \times 0.2 + 0.9999 \times 0.005 \times 0.01 + 0.0001 \times 0.5 \times 0.2 + 0.0001 \times 0.5) \times 34$$

≈ 0.03740 million

Suppose these events for each $1/4$ months are i.i.d.,

the total risk for 11 years is $11 \times 12 \times 4 \times 0.03740 \approx 19.74$ million.

Q7 The success / failure of manual detection is dependent on the event that whether the sensor detection works or not.

Q8. Considering the failure of the system is equivalent to the event (panel fails and at least $(n-1)$ batteries fail) or (panel works and (n) batteries fail),

the unreliability function for a system with one solar panel and (n) battery panels can be expressed as.

$$F_n(t) = F_{panel}(t) \times \prod_{i=1}^{n-1} F_{battery}(t) + (1-F_{panel}(t)) \times \prod_{i=1}^{n} F_{battery}(t).$$

$= (1-\exp(-\lambda_1 t)) \times (1-\exp(-\lambda_2 t))^{n-1} + \exp(-\lambda_1 t) \times (1-\exp(-\lambda_2 t))^{n}$.

The required reliability for 3 years. $R_n(t) = 1 - F_n(3 \times 365 \times 24 \text{ hours})$
$= 1 - F_n(26280 \text{ hours})$.

Substitute $\lambda_1 = 10^{-7}/hr$, $\lambda_2 = 10^{-5}/hr$, we can derive

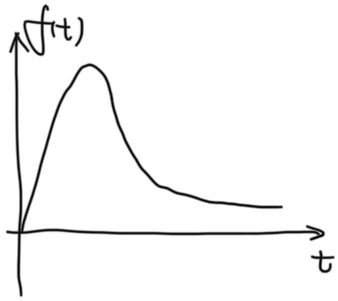
$$R_n = 1 - [(1-\exp(-2.628\times10^{-3})) \times (1-\exp(-0.2628))^{n-1} + \exp(-2.628\times10^{-3}) \times (1-\exp(-0.2628))^{n}]$$

$$\Leftrightarrow (1-\exp(-0.2628))^{n} \left(\exp(-2.628\times10^{-3} + \frac{1-\exp(-2.628\times10^{-3})}{1-\exp(-0.2628)}\right) = R_n \geq 2\times10^{-4}.$$

$$\Leftrightarrow n \geq 1.$$

Q9. One assumption is that the failed device will not recover in the time period of consideration.
Another assumption is that the failures of each device are independent.
If the 1st assumption does not hold, the calculated reliability is lower;
if the 2nd assumption does not hold, and the failure of one device may increase the failure prob. of other devices, the calculated reliability is higher.

Q10. a.



The hazard function $f(t) = -\frac{d}{dt} R(t)$
$= 2ct \cdot \exp(-ct^2)$

The curve goes up from $(0, 0)$, reaches the maximum at $t = \sqrt{\frac{1}{2c}}$, and then decreases to 0 as $t \to \infty$.

b. $\text{MTTF} = \int_0^\infty R(t)\,dt$
$= \int_0^\infty \exp(-ct^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{c}}$.

c. For a shifted exponential distribution with shift T, and occurrence rate ν.
the $\text{MTTR} = T + \frac{1}{\nu}$.
The unit for c is $[t]^{-2}$, for T is $[t]$, for ν is $[t]^{-1}$.
The inherent availability $A_I = \frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}} = \frac{\sqrt{\frac{\pi}{c}}}{\sqrt{\frac{\pi}{c}} + 2T + \frac{2}{\nu}}$.

d. The actual downtime during a certain time period is required.

Q11. Monte Carlo method can be used to estimate the prob. of bridge collapse.
It is a simulation-based method that is commonly adopted.
Main steps:
1. set the number of Monte Carlo simulations N, times of collapse $N_c \leftarrow 0$.
2. sample the 25 r.v.s, run the computation program to calculate the

maximum load

3. compare the calculated maximum load and the actual / required load, if the maximum load cannot meet the requirement, $N_c \leftarrow N_c + 1$
4. after N simulations, estimate the prob. of collapse $p_c \leftarrow \frac{N_c}{N}$.

Limitations:

Since the true prob. is very small (10^{-5}), N needs to be large ($>10^5$) so that the failure case is observed.

Q12. Since many engineering systems involve interactions with humans, and human decisions have consequences to the whole system, it's important to model human factors in risk management. Besides, the ultimate goal for technologies is to better serve humans, and thus humans must be placed at the core of risk management.

The difficulties in modeling human error are:

1. human behaviors vary by individual, the population of humans has a large variation;
2. human errors are not static, in other words, the model may change by time as the user learns from experience;
3. it's costly and laborious to sample a large amount data of user behaviors, besides, regulations and laws about user privacy may prevent the acquisition of certain user data.