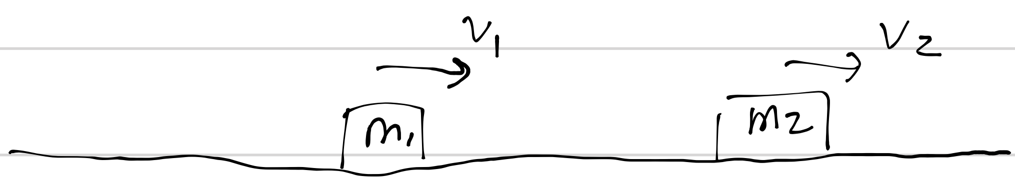
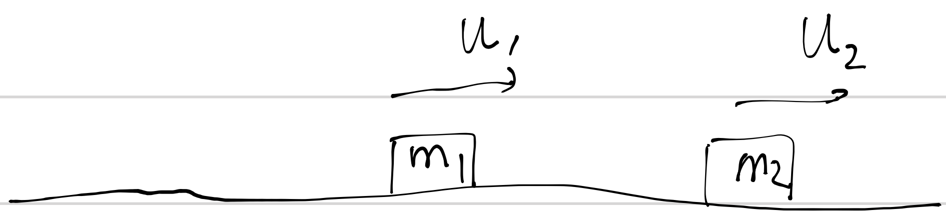
Before collision



After collision



(i) $\Delta v = v_2 - v_1$ $\qquad e = \frac{\Delta u}{\Delta v}$

$\Delta u = u_2 - u_1 = e\Delta v$

In the lab frame

$$k = k_1 + k_2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

Let's denote the velocity in the center of mass reference frame as

v_1' $\quad v_2'$

$v_1' = v_1 - v_{cm}$

$v_2' = v_2 - v_{cm}$

$M v_{cm} = m_1 v_1 + m_2 v_2 \qquad M = m_1 + m_2$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{M}$$

$K_{cm} = \frac{1}{2} M V_{cm}^2$

Energy related to the center of mass movement will not change during a collision

might change during a collision

$K_{relative\ motion} = \frac{1}{2} \mu V_{rel}^2$

μ is the reduced mass of the two trains

$\mu = \frac{m_1 m_2}{M}$

$\Delta u = u_2 - u_1$

$\Delta V = v_2 - v_1$

$\frac{\Delta u}{\Delta V} = e$

i) $\Delta E = \frac{1}{2}\mu(V_{rel,f}^2 - V_{rel,i}^2)$

$= \frac{1}{2}\mu(\Delta u^2 - \Delta V^2)$

$= \frac{1}{2}\mu(e^2 \Delta V^2 - \Delta V^2)$

$= \frac{1}{2}\mu \Delta V^2 (e^2 - 1)$

ii) when the two train cars comple together

$\Delta u = 0 \Rightarrow e = 0$

when $e = 0$

$\Delta E = -\frac{1}{2}\mu \Delta V^2$

iii) to achieve a smaller V_{min}

$|(e^2-1)|$ should be the largest

from the last problem, we know $|e^2-1|$ is largest when $e=0$

In such case

$$\Delta E = -\frac{1}{2}\mu \Delta V^2$$

thus $V_{min} = \sqrt{\frac{2\Delta E}{\mu}}$