

Problem 3

	$P = (C, T)$	$T = 0$	$T = 1$	$P(C)$
$C = 0$	$P(0,0) = P(0,0 X=0)P(X=0) + P(0,0 X=1)P(X=1)$ $= \frac{2}{5} \times \frac{1}{4} + \frac{3}{5} \times \frac{3}{4} = \frac{11}{20}$	$P(0,1) = P(0,1 X=0)P(X=0) + P(0,1 X=1)P(X=1)$ $= \frac{1}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{4}{20} = \frac{1}{5}$	$\frac{11}{20} + \frac{1}{5} = \frac{3}{4}$	
$C = 1$	$P(1,0) = P(1,0 X=0)P(X=0) + P(1,0 X=1)P(X=1)$ $= \frac{4}{15} \times \frac{1}{4} + \frac{3}{20} \times \frac{3}{4} = \frac{43}{240}$	$P(1,1) = P(1,1 X=0)P(X=0) + P(1,1 X=1)P(X=1)$ $= \frac{2}{15} \times \frac{1}{4} + \frac{1}{20} \times \frac{3}{4} = \frac{17}{240}$	$\frac{43}{240} + \frac{17}{240} = \frac{1}{4}$	
$P(T)$	$\frac{11}{20} + \frac{43}{240} = \frac{35}{48}$	$\frac{1}{5} + \frac{17}{240} = \frac{13}{48}$	1	

	$P(C, X)$	$X=0$	$X=1$	$P(C)$
$C=0$	$P(C=0, X=0) = P(C=0 X=0)P(X=0)$ $= \frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$	$P(C=0, X=1) = P(C=0 X=1)P(X=1)$ $= \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$	$\frac{3}{20} + \frac{3}{5} = \frac{3}{4}$	
$C=1$	$P(C=1, X=0) = P(C=1 X=0)P(X=0)$ $= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$	$P(C=1, X=1) = P(C=1 X=1)P(X=1)$ $= \frac{1}{5} \times \frac{3}{4} = \frac{3}{20}$	$\frac{1}{10} + \frac{3}{20} = \frac{1}{4}$	
$P(X)$	$\frac{3}{20} + \frac{1}{10} = \frac{1}{4}$	$\frac{3}{5} + \frac{3}{20} = \frac{3}{4}$	1	

	$P(X, T)$	$T=0$	$T=1$	$P(X)$
$X=0$	$P(X=0, T=0) = P(T=0 X=0)P(X=0)$ $= \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$	$P(X=0, T=1) = P(T=1 X=0)P(X=0)$ $= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$	$\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$	
$X=1$	$P(X=1, T=0) = P(T=0 X=1)P(X=1)$ $= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$	$P(X=1, T=1) = P(T=1 X=1)P(X=1)$ $= \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$	$\frac{9}{16} + \frac{3}{16} = \frac{3}{4}$	
$P(T)$	$\frac{1}{6} + \frac{9}{16} = \frac{35}{48}$	$\frac{1}{12} + \frac{3}{16} = \frac{13}{48}$	1	

2. Check X and T

$$P(X=0, T=0) = \frac{1}{6} \neq P(X=0) \times P(T=0) = \frac{1}{4} \times \frac{35}{48}$$

X and T are not independent

Check X and C

$$P(X=0, C=0) = \frac{3}{20} \neq P(X=0)P(C=0) = \frac{1}{4} \times \frac{3}{4}$$

X and C are not independent

Check C and T

$$P(C=0, T=0) = \frac{11}{20} \neq P(C=0)P(T=0) = \frac{1}{4} \times \frac{35}{48}$$

C and T are not independent

Therefore no pair of variables are independent.

3. Check $C=0, T=0$

$$P(C=0, T=0 | X=0) = \frac{2}{5} \quad (\text{from the left table})$$

$$\begin{aligned} & P(C=0 | X=0) \cdot P(T=0 | X=0) \\ &= \frac{3}{5} \times \frac{2}{3} = \frac{2}{5} \end{aligned}$$

Check other values of C and T

$$P(C=0, \bar{T}=1 | X=0) = \frac{1}{5} = P(C=0 | X=0) \cdot P(\bar{T}=1 | X=0) = \frac{3}{5} \times \frac{1}{3}$$

$$P(C=1, T=0 | X=0) = \frac{4}{15} = P(C=1 | X=0) \cdot P(T=0 | X=0) = \frac{2}{5} \times \frac{2}{3}$$

$$P(C=1, \bar{T}=1 | X=0) = \frac{2}{15} = P(C=1 | X=0) \cdot P(\bar{T}=1 | X=0) = \frac{2}{5} \times \frac{1}{3}$$

For $X=1$

$$P(C=0, T=0 | X=1) = \frac{3}{5} = P(C=0 | X=1) \cdot P(T=0 | X=1) = \frac{4}{5} \times \frac{3}{4}$$

$$P(C=0, \bar{T}=1 | X=1) = \frac{1}{5} = P(C=0 | X=1) \cdot P(\bar{T}=1 | X=1) = \frac{4}{5} \times \frac{1}{4}$$

$$P(C=1, T=0 | X=1) = \frac{3}{20} = P(C=1 | X=1) \cdot P(T=0 | X=1) = \frac{1}{5} \times \frac{3}{4}$$

$$P(C=1, \bar{T}=1 | X=1) = \frac{1}{20} = P(C=1 | X=1) \cdot P(\bar{T}=1 | X=1) = \frac{1}{5} \times \frac{1}{4}$$

Therefore, C and T are conditionally independent, given X

4.

For $\bar{T}=0$

$$P(C=0, X=0 | \bar{T}=0) = \frac{P(C=0, X=0, \bar{T}=0)}{P(\bar{T}=0)} = \frac{P(C=0, \bar{T}=0 | X=0) P(X=0)}{P(\bar{T}=0)}$$

$$= \frac{\frac{2}{5} \times \frac{1}{4}}{\frac{35}{48}} = \frac{24}{175}$$

$$P(C=0 | \bar{T}=0) = \frac{P(C=0, \bar{T}=0)}{P(\bar{T}=0)} = \frac{\frac{11}{20}}{\frac{35}{48}} = \frac{132}{175}$$

$$P(X=0 | \bar{T}=0) = \frac{P(X=0, \bar{T}=0)}{P(\bar{T}=0)} = \frac{\frac{1}{6}}{\frac{35}{48}} = \frac{8}{35}$$

$$P(C=0, X=0 | T=0) \neq P(C=0 | T=0) \cdot P(X=0 | T=0)$$

Therefore C and X are **not**
conditionally independent.