a)

Interarrival time $X \sim exp(\lambda)$

$$EX = \frac{1}{\lambda} = \frac{1}{10} h = 6 \text{ mins.}$$

b)

Length $\tau = \frac{1}{3}$ hours.

Thus $X \sim$ possion $(\frac{10}{3})$.

$$EX = \frac{10}{3}$$

c)

The interarrival time $X \sim exp(\lambda)$

$$p(X > \frac{1}{6} + \frac{1}{6} \mid X > \frac{1}{6}) = p(X > \frac{1}{6})$$

$$= e^{-\frac{\lambda}{6}}$$

$$= e^{-\frac{5}{3}}$$

d)

Let X denote the number of arrivals between 7:20 pm and 7:30 pm.

Then

$$X \sim \text{possion}\left(\frac{\lambda}{6}\right)$$

Since possion process has independent increments,

$$p(X=2 \mid \text{Last penguin arrivals at 7:10pm})$$

$$= p(X=2) = \frac{e^{-\frac{\lambda}{6}} \cdot \lambda^2}{2!}$$

$$= \frac{e^{-\frac{5}{3}} \cdot \left(\frac{5}{3}\right)^2}{2}$$

$$= 0.2623$$

e)

Let X denote the number of arrival between 7 pm and 7:20 pm

Let Y donote the number of arrival

between 7:20 pm and 7:40 pm.

Then

$$P(X=3 \mid X+Y=5) = \frac{P(X=3,\ X+Y=5)}{P(X+Y=5)}$$

$$= \frac{P(X=3,\ Y=2)}{P(X+Y=5)}$$

$$= \frac{P(X=3)\,P(Y=2)}{P(X+Y=5)}$$

$X \sim$ possion$\left(\frac{10}{3}\right)$

$X+Y \sim$ possion (10)

$Y \sim$ possiom$\left(\frac{20}{3}\right)$

$$P(X=3 \mid X+Y=5) = \frac{\dfrac{e^{-\frac{10}{3}} \cdot \left(\frac{10}{3}\right)^3}{3!} \cdot \dfrac{e^{-\frac{20}{3}} \cdot \left(\frac{20}{3}\right)^2}{2!}}{e^{-10} \cdot 10^5 / 5!}$$

$$= \frac{5!}{3!\,2!} \cdot \frac{10^3 \cdot 20^2}{10^5} \cdot \frac{1}{3^5}$$

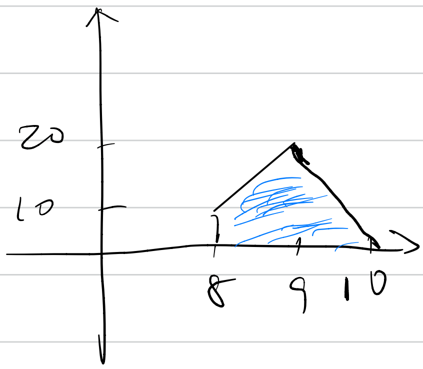
$$= 10 \cdot \frac{2^2}{3^5} \doteq 0.1646$$

(f)

Let X denote the number of arrival between 8 pm and 10 pm.

$X \sim$ possion (μ) with

$$\mu = 15 + 10 = 25.$$



Thus

$$P(X=30) = \frac{e^{-25} \cdot 25^{30}}{30!} = 0.0454.$$