Exercise 1

a) $y = \dfrac{x^2-3}{x-2} = \dfrac{x^2-4+1}{x-2} = x+2+\dfrac{1}{x-2}$

$y' = 1-(x-2)^{-2}$

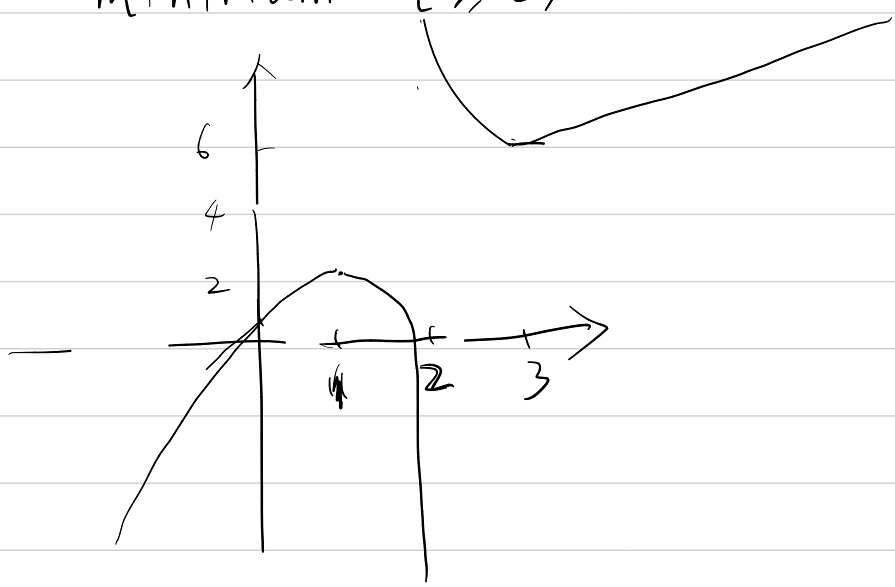
$y'' = 2(x-2)^{-3}$

$y' = 0 \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 3 \end{cases}$

Local Maxima $(1, 2)$

no inflection point

Local minimum $(3, 6)$



b) $y = e^{x} - 2e^{-x} - 3x$

$y' = e^{x} + 2e^{-x} - 3$

$-lg^{2}$

$2 - 2 \cdot \frac{1}{2}$

$y'' = e^{x} - 2e^{-x}$

$1 - 3x$

$y' = 0 \qquad e^{x} + 2e^{-x} - 3 = 0$

let $a = e^{x} \qquad a > 0$

$a + \frac{2}{a} - 3 = 0$

$a^{2} + 2 - 3a = 0$

$a_1 = 1$

$a_2 = 2$

$e^{x} = 1 \qquad x = 0$

$e^{x} = 2 \qquad x = \ln 2$

$y'' = 0 \qquad e^{x} = 2e^{-x} \qquad e^{2x} = 2$

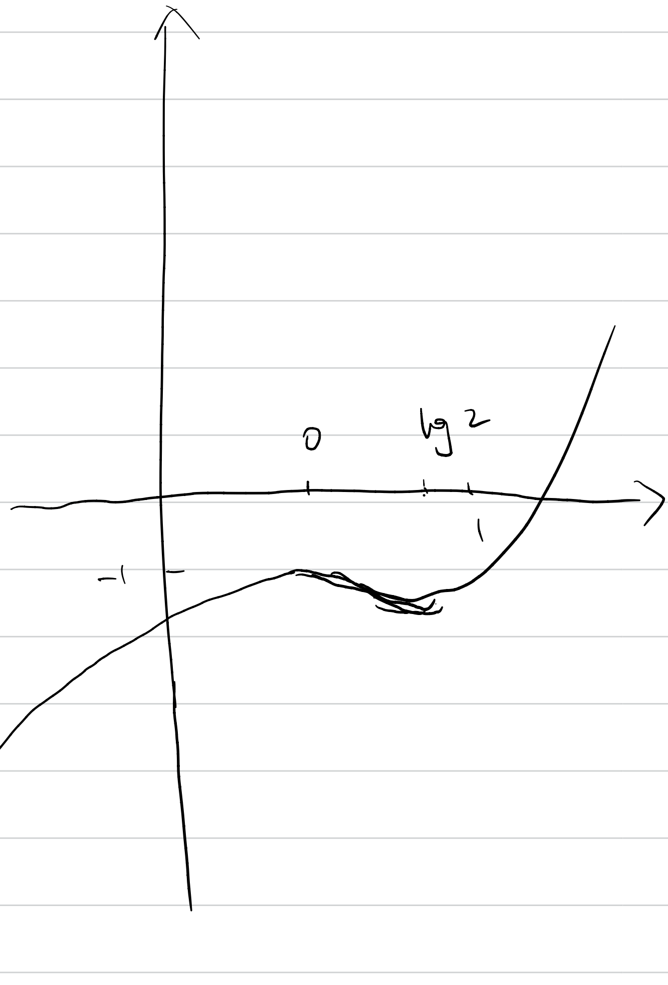
$2x = \ln 2$

$x = \frac{\ln 2}{2} = \ln\sqrt{2}$

Local maxima $(0, -1)$

Local minima $(\ln 2, 1-3\ln 2)$

Infection points $(\lg \sqrt{2}, -3\ln\sqrt{2})$



c)

$y = \ln(\cos x)$ we graph it at $-\frac{\pi}{2} < x < \frac{\pi}{2}$

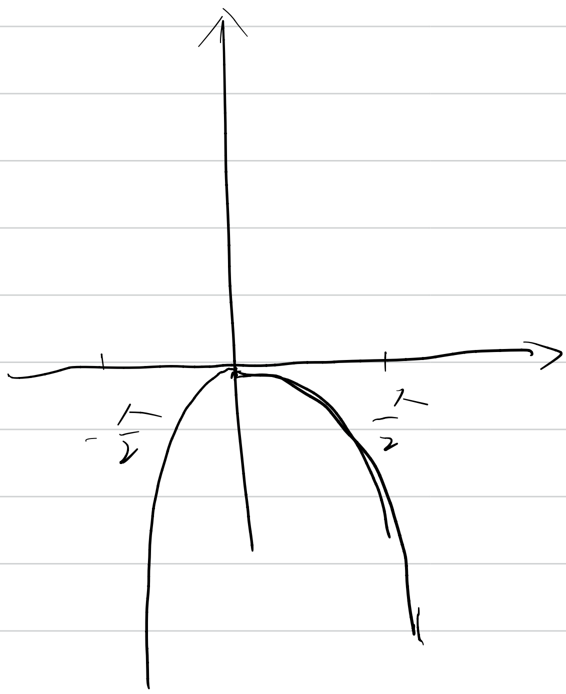
$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$

$y'' = -\left(\frac{\sin x}{\cos x}\right)' = -\frac{\cos^2 + \sin^2 x}{\cos^2 x} = -\frac{1}{\cos^2 x}$

$y' = 0$ $\tan x = 0$ $x = 0$

Local maxima $(0, 0)$

no infection points



Exercise 2

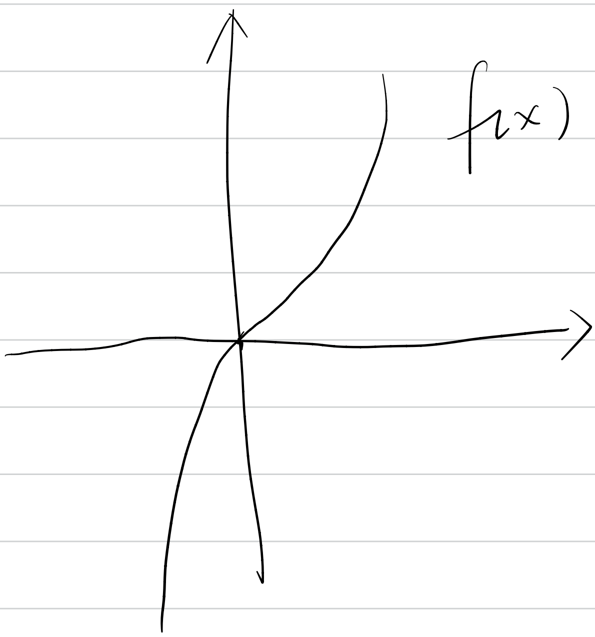
a)

$$y''' = \begin{cases} -2x & x \leq 0 \\ 2x & x > 0 \end{cases}$$

$y' = 0$, $x = 0$ critical point

For $x > 0$, curve is increasing

$x < 0$, curve is decreasing



$(x^2-2x)(x^2-10x+25)$

$x^4-12x^3+45x-50x$

b)

$y' = (2x-2)(x-5)^2 + 2(x-5)(x^2-2x)$

$= 2(x-5)[(x-1)(x-5) + x^2-2x]$

$= 2(x-5)[2x^2-8x+5]$

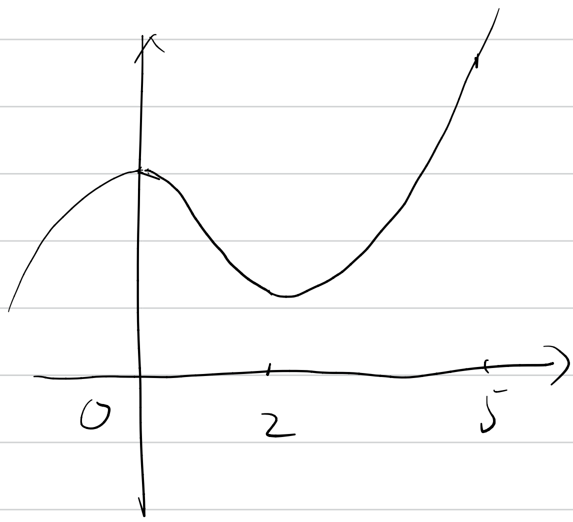
$y'=0$ $\quad x_1=0, \quad x_2=2, \quad x_3=5$

$y''<0 \quad y''>0 \quad 0$

For $x<0$, $\quad y'>0$

$0<x<2 \quad y'<0$

$x \geq 2 \quad y' \geq 0$



c)

$$y' = 1 - \cot^2\theta = 1 - \frac{\cos^2\theta}{\sin^2\theta}$$

$$y'' = -2\cot\theta \cdot \left(-\frac{1}{\sin^2\theta}\right)$$

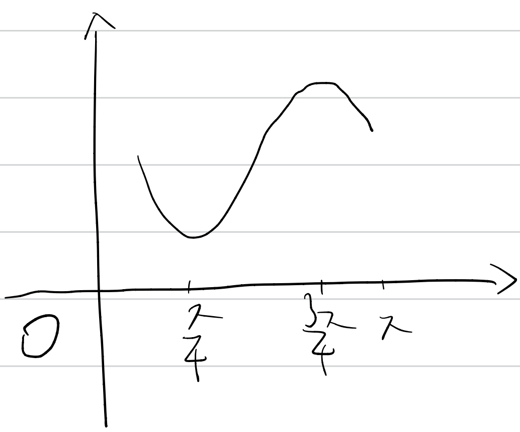
$$= 2 \cdot \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin^2\theta}$$

$$= \frac{2\cos\theta}{\sin^3\theta}$$

$y' = 0$ $\quad \tan\theta = \pm 1$ $\quad$ For $\theta < \frac{\pi}{4}$ $\quad y' < 0$

$\theta_1 = \frac{\pi}{4}$ $\quad \theta_2 = \frac{3}{4}\pi$ $\quad$ For $\frac{\pi}{4} < \theta < \frac{3}{4}\pi$ $\quad y' > 0$

$y'' > 0$. $y'' < 0$ $\quad$ For $\theta > \frac{3}{4}\pi$ $\quad y' < 0$



Exercise 3

$$y = \frac{4x}{x^2+4}$$

1° Domain $\quad x \in \mathbb{R}$

The curve is symmetric about 0.

$$y(-x) = -y(x)$$

2° $$y' = \frac{4(x^2+4) - 4x(2x)}{(x^2+4)^2} = \frac{-4x^2+16}{(x^2+4)^2}$$

$$y'' = \frac{-8x(x^2+4)^2 - (-4x^2+16)\,2(x^2+4)\,2x}{(x^2+4)^4}$$

$$= \frac{-8x(x^2+4) - 4x(-4x^2+16)}{(x^2+4)^3}$$

$$= \frac{-4x}{(x^2+4)^3}\left[2x^2+8-4x^2+16\right]$$

$$= \frac{-4x}{(x^2+4)^3}(-2x^2+24) = \frac{8x(x^2-12)}{(x^2+4)^3}$$

3° critical

$y' = 0 \qquad 16 - 4x^2 = 0$

$x_1 = 2 \qquad x_2 = -2$

$y'' > 0 \qquad y'' < 0$

4° For $x < -2$, $\quad y' < 0$

$-2 < x < 2 \qquad y' > 0$

$x > 2 \qquad y' < 0$

5° $y'' = 0 \qquad \therefore x_1 = -\sqrt{12}$

$x_2 = 0$

$x_3 = \sqrt{12}$

$x < -\sqrt{12}$, concave up

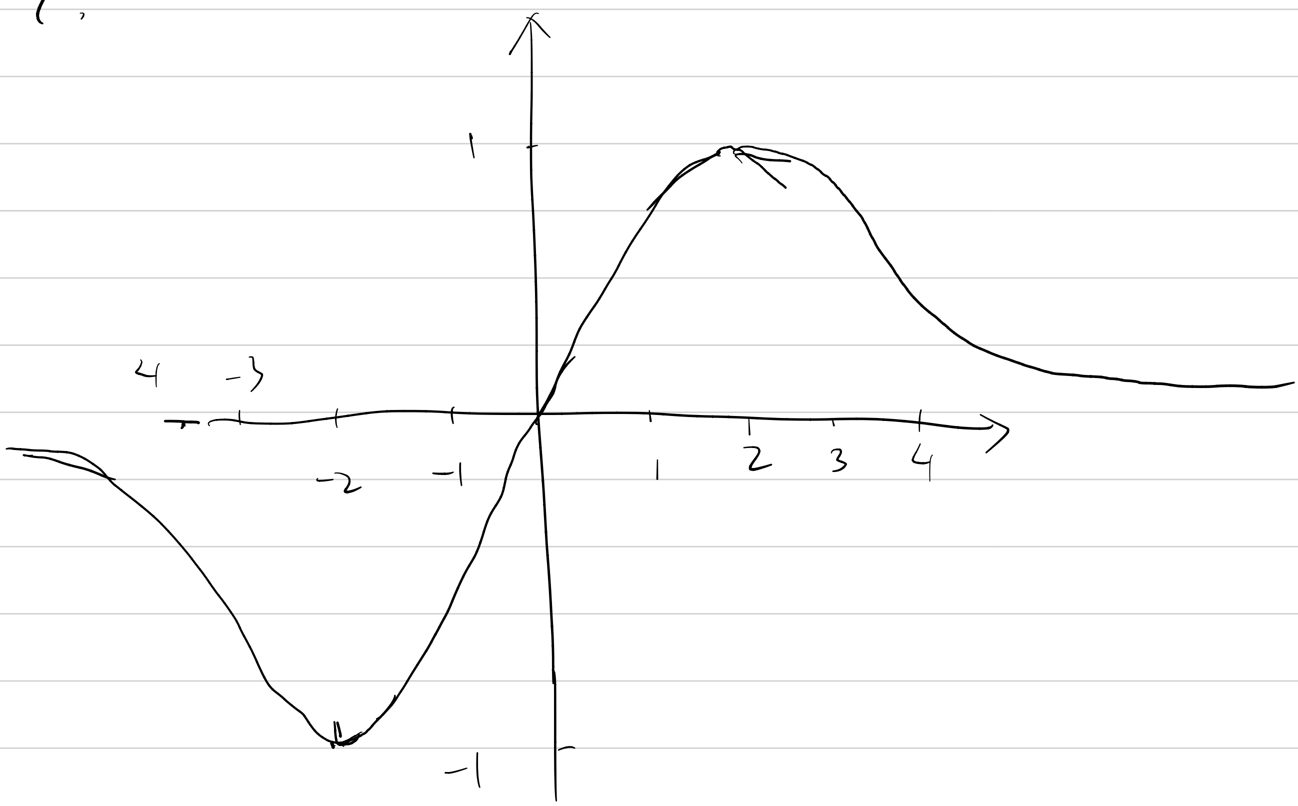
$-\sqrt{12} < x < 0$ concave down

$0 < x < \sqrt{12}$ concave up

$x > \sqrt{12}$ concave down

6. No asymptotes

7.



Exercise 4

$$y = \frac{x^2+a}{bx+c}$$

$$y' = \frac{2x(bx+c) - b(x^2+a)}{(bx+c)^2}$$

$$= \frac{bx^2+2cx-ab}{(bx+c)^2}$$

Local Maximum at $(-1, -2)$.

So $\frac{1+a}{c-b} = -2 \Rightarrow 2b-2c = 1+a$. ①

$y'(-1) = 0$

$y'(3) = 0$

$$\Rightarrow \begin{cases} \frac{b-2c-ab}{(c-b)^2} = 0 \\ \frac{9b+6c-ab}{(c+3b)^2} = 0 \end{cases} \Rightarrow \begin{cases} b-2c-ab=0 \quad ② \\ 9b+6c-ab=0 \quad ③ \end{cases}$$

$$\begin{cases} 2b-2c = 1+a & ① \\ b-2c-ab = 0 & ② \\ 9b+6c-ab=0 & ③ \end{cases}$$

$③ - ② \Rightarrow 8b+8c=0 \Rightarrow b=-c \quad ④$

$④$ and $① \Rightarrow a = 4b-1. \quad ⑤$

Substitue $④, ⑤$ into $③$,

$4b(1-b)=0 \qquad b=0$ or $b=1$.

If $b=0 \quad c=0 \quad a=-1$, this is imposible

So $b=1 \quad c=-1 \quad a=3$

$$y = \frac{x^2+3}{x-1}$$