Excercise 1

a) $$y^2 = \sqrt{\frac{1+x}{1-x}} = \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$$

$$2y \cdot \frac{dy}{dx} = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}(1-x)^{\frac{1}{2}} + \frac{1}{2}(1-x)^{-\frac{1}{2}}(1+x)^{\frac{1}{2}}}{1-x}$$

$$= \frac{\frac{1}{2}(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{1}{2}}}{1-x}(1+x+1-x)$$

$$= (1+x)^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}y^{-1}(1+x)^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$$

$$= \begin{cases} \frac{1}{2}(1-x)^{-\frac{5}{4}}(1+x)^{-\frac{3}{4}} & 于\ y>0 \\ -\frac{1}{2}(1-x)^{-\frac{5}{4}}(1+x)^{-\frac{3}{4}} & 于\ y<0 \end{cases}$$

b)

$$y\, e^{\tan^{-1} x} = 2$$

$$\log y + \tan^{-1} x = 0$$

$$\frac{1}{y} \cdot \frac{dy}{dx} - \frac{1}{\sin^2 x} = 0$$

$$\frac{dy}{dx} = \frac{y}{\sin^2 x}$$

$$= \frac{2}{\sin^2 x} \cdot e^{-\tan^{-1} x}$$

Exercise 2.

$$y^3 + y = 2\cos x$$

$$3y^2 \cdot \frac{dy}{dx} + \frac{dy}{dx} = -2\sin x \qquad (1)$$

$$3\left[2y \cdot \frac{dy}{dx} + y^2 \cdot \frac{dy^2}{dx^2}\right] + \frac{dy^2}{dx^2} = -2\cos x. \quad (2)$$

At point $(0,1)$, by (1),

$$3 \cdot \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

By (2).

$$3\left[0 + \frac{dy^2}{dx^2}\right] + \frac{dy^2}{dx^2} = -2$$

$$4\frac{dy^2}{dx^2} = -2$$

$$\frac{dy^2}{dx^2} = -\frac{1}{2}$$

Excercise 3

The slope for the normal line is

$$-\frac{1}{f'(x)} = -\frac{1}{2}$$

So $f'(x) = 2$

$y = \tan x$

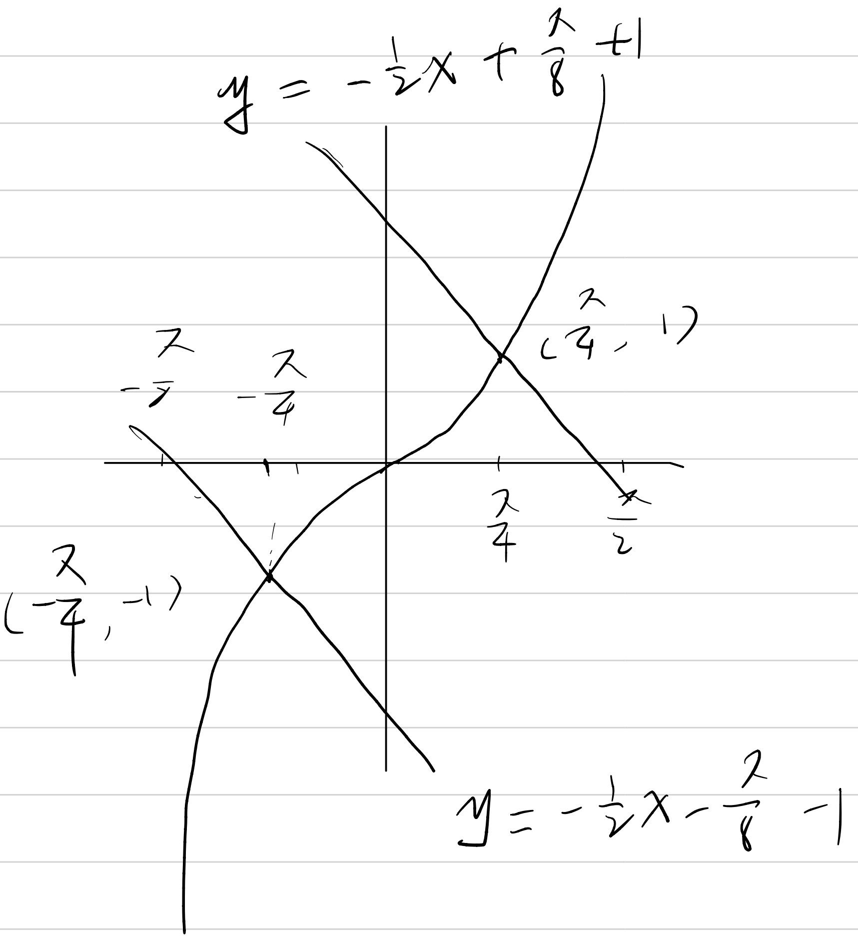
$$\frac{dy}{dx} = \frac{1}{\cos^2 x} = 2$$

$$\cos^2 x = \frac{1}{2}$$

Since $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \pm\frac{\pi}{4}.$$


$y = -\frac{1}{2}x + \frac{\pi}{8} + 1$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
$(\frac{\pi}{4}, 1)$
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$(-\frac{\pi}{4}, -1)$
$y = -\frac{1}{2}x - \frac{\pi}{8} - 1$

Excercise 4

$$\frac{1-2\cot^2\theta}{5\cot^2\theta-7\cot\theta-8}$$

$$=\frac{1-2\cdot\frac{\cos^2\theta}{\sin^2\theta}}{5\frac{\cos^2\theta}{\sin^2\theta}-7\frac{\cos\theta}{\sin\theta}-8}$$

$$=\frac{\sin^2\theta-2\cos^2\theta}{5\cos^2\theta-7\cos\theta\sin\theta-8\sin^2\theta}$$

$$\lim_{\theta\to0^+}\frac{1-2\cot^2\theta}{5\cot^2\theta-7\cot\theta-8}$$

$$=\lim_{\theta\to0^+}\frac{\sin^2\theta-2\cos^2\theta}{5\cos^2\theta-7\cos\theta\sin\theta-8\sin^2\theta}$$

$$=-\frac{2}{5}$$

Exercise 5

$f(0) = \sqrt{1+0} + \sin 0 - 0.5 = 1 - 0.5 = 0.5$

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} + \cos x$.

$f'(0) = \frac{1}{2} + 1 = \frac{3}{2}$

So the linearlization of $f(x)$ at $x=0$

is

$y = f(0) + f'(0) \cdot x$

$= 0.5 + 1.5x$

Exercise 6.

Note that $g(y) = 2e^{y}$ is a monotone function of y.

Moreover, $-1 \leq \sin\frac{x}{2} \leq 1$.

Thus

$$\max_x f(x) = 2e^1 = 2e$$

$$\min_x f(x) = 2e^{-1} = 2/e.$$

When $\sin\frac{x}{2} = 1$, we have $\frac{x}{2} = \frac{\pi}{2} \pm 2k\pi$

$$x = \pi \pm 4k\pi$$

When $\sin\frac{x}{2} = -1$, we have $\frac{x}{2} = -\frac{\pi}{2} \pm 2k\pi$

$$x = -\pi \pm 4k\pi.$$

To conclude,

when $x = \pi \pm 4k\pi$, where k is an integer,

$f(x)$ takes its maximum, the

maximum value is $2e$.

when $x = -\pi \pm 4k\pi$, where k is an integer,

$f(x)$ takes its minimum, the

minimum value is $\frac{2}{e}$.