6.

a) b is called damping coeftient
Damping has the effect of reducing or preventing its oscilation.

b) $b > 0$

c) the general solution is

$$\underbrace{c_1 e^{st} \cos mt}_{y_1(t)} + c_2 e^{st} \sin mt$$

$$y_1(t)' = e^{st} c_1 [s\cos(mt) - m\sin(mt)]$$

$$y_1(t)'' = e^{st} c_1 [(-m^2+s^2)\cos mt - 2ms \sin mt]$$

$$y_1(t)'' + 4y_1(t') + 13y = 0$$

$$e^{st} c_1 [(-m^2+s^2)\cos mt - 2ms \sin mt] + 4e^{st} c_1 [s\cos(mt) - m\sin(mt)]$$
$$+ 13 e^{st} c_1 \cos mt = 0$$

divided by $e^{st} c_1$ on both sides, we obtain

$$(-m^2+s^2)\cos mt - 2ms \sin mt + 4s\cos(mt) - 4m\sin(mt)$$

$+13\cos mt = 0$

$$\Rightarrow \begin{cases} (-m^2+s^2)+4s+13=0 \\ -2ms-4m=0 \end{cases} \Rightarrow \begin{cases} m=3 \\ s=-2 \end{cases}$$

Therefore the general solution is

$$c_1 e^{-2t}\cos(3t) + c_2 e^{-2t}\sin(3t)$$

d) $y(0)=0$

$y'(0) = 10 \text{ cm/s}$

e) i. $f(t) = 2\cos 3t$

$$\{A\cos 3t,\ B\sin 3t\}$$

ii. $f(t) = e^{-2t}\sin 3t$

$$\{e^{-2t}\sin 3t,\ e^{-2t}\cos 3t\}$$

f) $$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} + 4v + 13y = f(t)$$

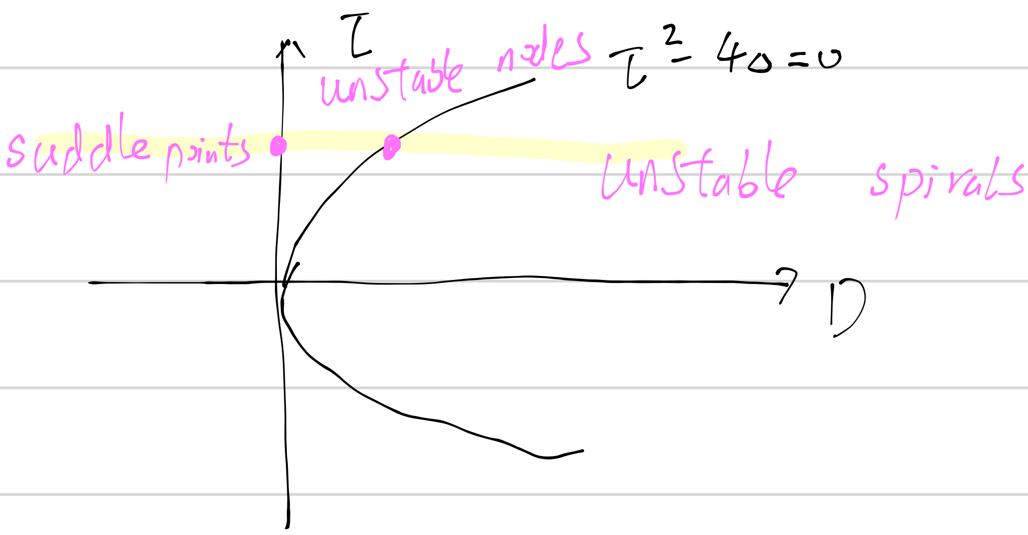
3.

a) $Y = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & k \\ 1 & 2 \end{bmatrix}$

b) determinant $= D = 2-k$

trace $= \tau = 2$

trace is always positive.



$D=0 \rightarrow$ bifurcation $\Rightarrow k=2$

$\tau^2 - 4D = 0 \rightarrow$ bifurcation $\Rightarrow k=1$

c) For $D<0$ $\quad 2-k<0$ $\quad k>2$

saddle $\left(\begin{array}{c} \lambda_1 \cdot \lambda_2 < 0 \\ \lambda_1, \lambda_2 \text{ real} \end{array} \right)$

For $0 < D < 1$ $\quad 1 < k < 2$

node $\quad (\lambda_1 \cdot \lambda_2 > 0)$

For $D > 1$ $\quad k < 1$

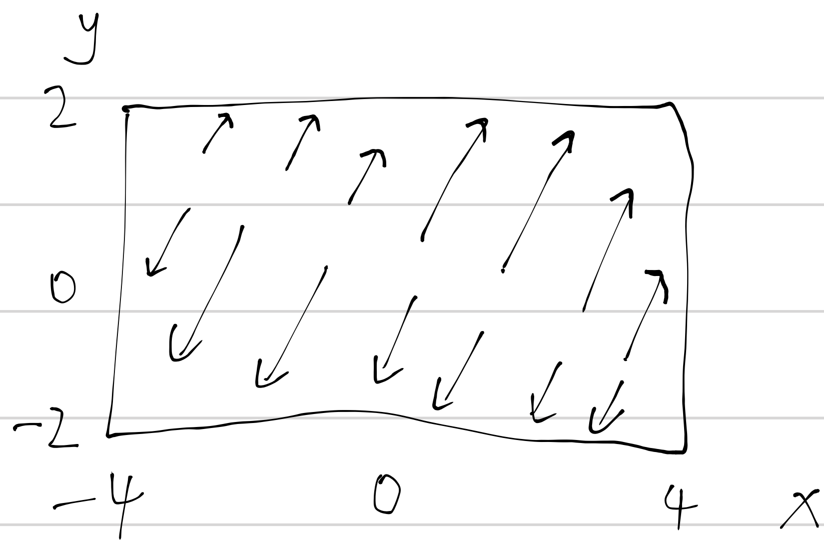
Focus (λ_1, λ_2 are complex numbers)

d) We only need to care about the solution with a positive eigenvalue

$\lambda_2 = 3 \quad v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

general solution is $\quad C_1 e^{3t} + C_2 e^{3t}$

phase portraint



If $x_0 + 2y_0 > 0$, at $t \to \infty$, $x \to \infty$, $y \to \infty$

at $t \to -\infty$, $x \to -\infty$ $y \to -\infty$

If $x_0 + 2y_0 < 0$, at $t \to \infty$, $x \to -\infty$, $y \to -\infty$

at $t \to -\infty$, $x \to \infty$, $y \to \infty$

x_0 and y_0 are intinal values for x and y

2.

a)

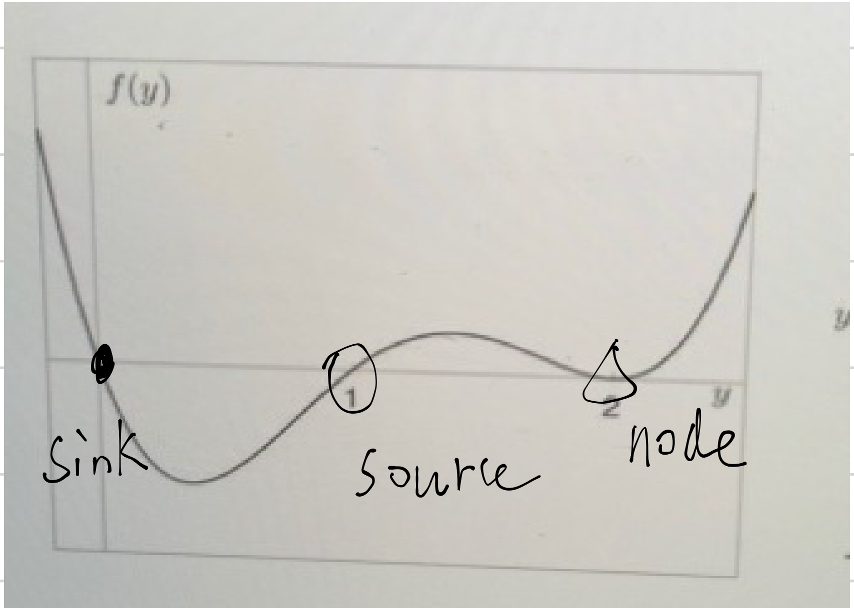
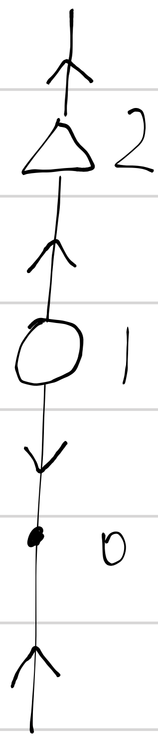


Figure 1



b)

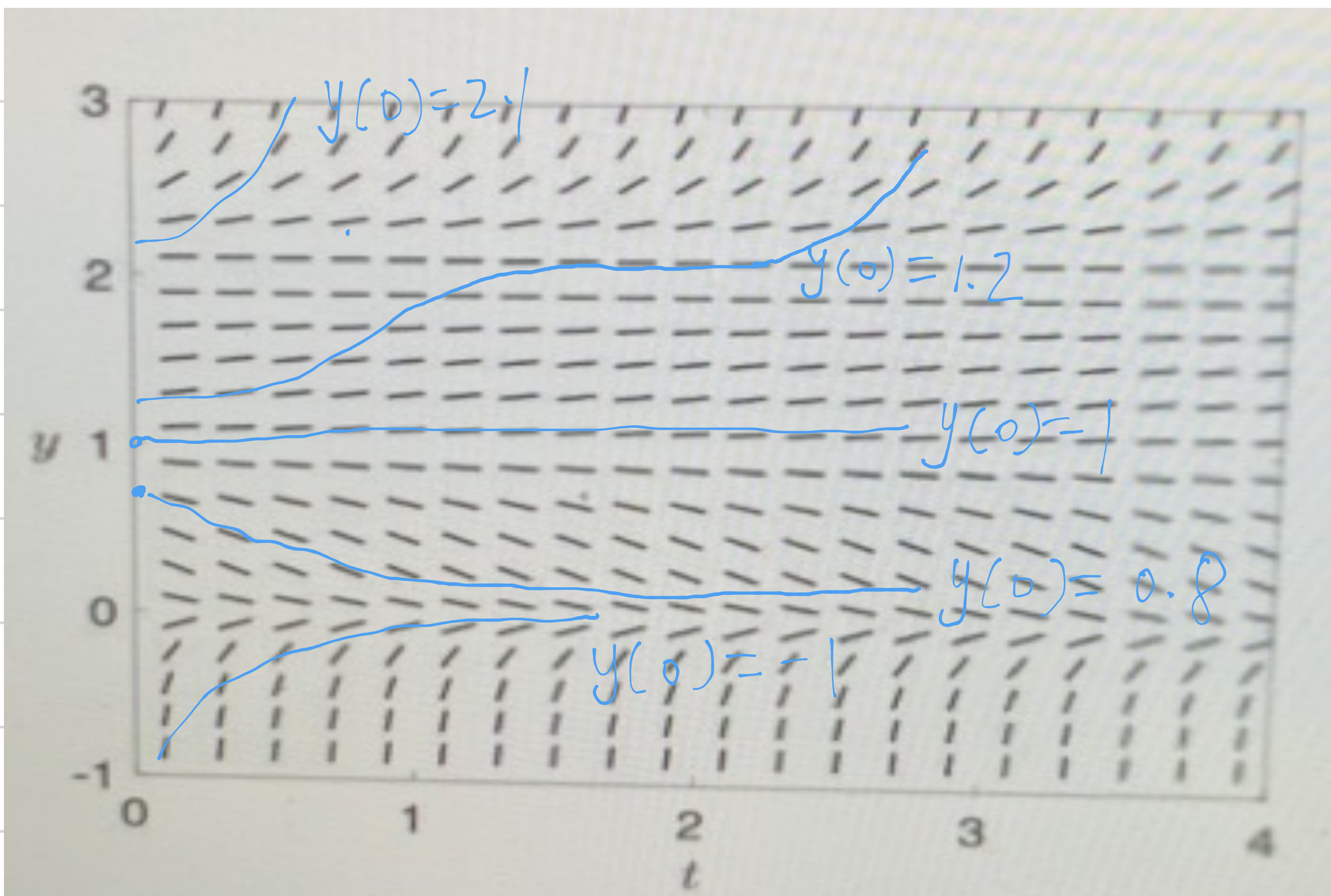


Figure 2

c. f(y) has 3 solutions

$y_1 = 0$ $y_2 = 1$ $y_3 = 2$ and opens up

thus, it could be a fourth order polynomial

$$g(y) = (y-2)^2$$

$$f(y) = y(y-1)(y-2)^2$$