(a)

Note that

$$|\sin^n(x)| \leq 1 \quad \text{for all } x \in [-1, 1].$$

Thus, by the dominated convergee theorem.

$$\lim_{n\to\infty} \int_{-\pi}^{\pi} \sin^n(x)\, d\lambda(x) = \int_{-\pi}^{\pi} \lim_{n\to\infty} \sin^n(x)\, d\lambda(x)$$

Note that

$$\lim_{n\to\infty} \sin^n(x) = 0, \quad \text{if } x \neq \pm 1.$$

Thus

$$\int_{-\pi}^{\pi} \lim_{n\to\infty} \sin^n(x)\, d\lambda(x) = 0.$$

(b)

$$\int_1^n \left(1-\frac{x}{2n}\right)^n d\lambda(x) = -\frac{2n}{n+1}\left(1-\frac{x}{2n}\right)^{n+1}\Bigg|_1^n$$

$$= \frac{2n}{n+1}\left[\left(1-\frac{1}{2n}\right)^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right]$$

Thus

$$\lim_{n\to\infty} \int_1^n \left(1-\frac{x}{2n}\right)^n d\lambda(x)$$

$$= \lim_{n\to\infty} \frac{2n}{n+1}\left(1-\frac{1}{2n}\right)^{n+1} - \lim_{n\to\infty} \frac{2n}{n+1}\cdot\left(\frac{1}{2}\right)^{n+1}$$

$$= 2\cdot e^{-\frac{1}{2}}$$