a)

Time	CF	PV of CF	Time weight PV of CF (Time x PV of CF)
1	5	$5/1.05 = 4.76$	4.76
2	5	$5/1.05^2 = 4.54$	9.08
3	100+5	$105/1.05^3 = 90.7$	272.10
Total		100	285.94

Macaulay duration = $\dfrac{\text{Total time weight pv of CF}}{\text{bond price}}$

YTM = coupon rate

Face value = current price.

Macaulay duration = $\dfrac{285.94}{100} = 2.86$ years.

Macaulay duration of a zero-coupon bond is 3 years, because there is no payment before bond matury. A zero-coupon bond ~~with coup~~ duration equals its maturity.

b)

modified duration = macaulay duration $/ (1 + \frac{YTM}{m})$

$= 2.86 / (1 + 0.05/1) = 2.72$ years.

$\Delta P = -D_{mod} \times \Delta i \times P$

$= -2.72 \times 0.1\% \times 100 = \cancel{2.72}\ -0.272$

price of the bond resulting from an increase in yield of 0.1% is 99.73.

$\Delta P = -2.72 \times 10\% \times 100 = 27.2$

price of the bond resulting from an increase in yield of 10% is 72.8.

The relation between YTM and bond price is negative. The YTM is increase will lead the bond price decrease.

c) Bond B is a zero cupon bond, So it's macaulay duration is equal to it maturity. Macaulay duration of bond B is 5 years.

d Immunization weight

$$\begin{cases} 4 = 2.86 W_A + 5 W_B \\ W_A = 1 - W_B \end{cases}$$

$W_A = 0.4673$ (weight in bond A) $W_B = 0.5327$ (weight in bond B)

PV of liability $= 10000 / 1.05^4 = 8227.07$

Bond A $0.4673 \times 8227.07 = 3844.51 \approx 40$ bond A

Bond B $0.5327 \times 8227.07 = 4382.56 \approx$ ~~44 bond A~~. 59 bond B.

The immunization portfolio needs 3844.51 bond A and 4382.56 bond B.
This portfolio should be rebalanced based on the interest rate change.