SIR Model:

$$\frac{dS}{dt} = -\beta SI,$$

$$\frac{dI}{dt} = \beta SI - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

where S is the susceptible; I is the infected; R is the recovered, β is the transmission parameter and γ is the recovery parameter.

At $t = 0, S(0) = N - 1, I(0) = 1, R(0) = 0$, where N is the population, i.e., 5×10^6.

a.

$$\gamma = \frac{1}{7}.$$

The reproduction number $R_0 = \frac{\beta N}{\gamma} = 4.5$, thus

$$\beta = \frac{R_0 \gamma}{N} = \frac{4.5 \times \frac{1}{7}}{5 \times 10^6} = 1.29 \times 10^{-7}.$$

b.

When the epidemic peaks, i.e., $\frac{dI}{dt} = 0$,

$$S_m = \frac{\gamma}{\beta} = \frac{N}{R_0} = \frac{5 \times 10^6}{4.5} = 1.11 \times 10^6,$$

$$I_m = -S_m + N + S_m \ln \frac{S_m}{S(0)} = 2.22 \times 10^6.$$

c.

$$\frac{dS}{dR} = -\frac{\beta SI}{\gamma I} = -\frac{\beta}{\gamma} S,$$

$$S(R) = S(0) e^{\frac{-\beta}{\gamma} R}.$$

As $t \to \infty, R \to R_\infty, I \to 0$,

$$S \to S_\infty = N - R_\infty,$$

$$S_\infty = S(0) e^{\frac{-\beta}{\gamma}(N - S_\infty)},$$

$$S_\infty = 58550.$$

d.

β is determined by the chance of contact and the probability of disease transmission. Considering that isolation reduces the chance of contact, we assume β is reduced by 30%.

e.

$$R_0 = \frac{\beta N}{\gamma} = \frac{1.29 \times 10^{-7} \times 0.7 \times 5 \times 10^6}{\frac{1}{3}} = 1.35.$$

f.

When the epidemic peaks, i.e., $\frac{dI}{dt} = 0$,

$$S_m = \frac{N}{R_0} = \frac{5 \times 10^6}{1.35} = 3.70 \times 10^6,$$

$$I_m = -S_m + N + S_m \ln \frac{S_m}{S(0)} = 1.9 \times 10^5.$$

g.

$$S_\infty = S(0)e^{\frac{-\beta}{\gamma}(N-S_\infty)} = S(0)e^{\frac{-R_0}{N}(N-S_\infty)},$$

$$S_\infty = 2653530.$$