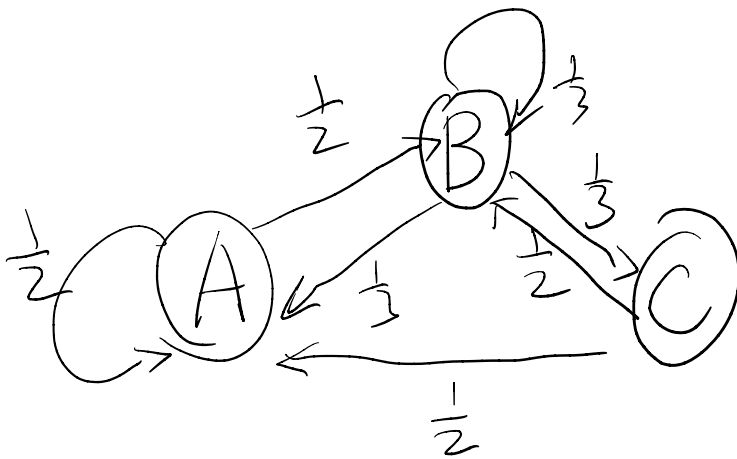


(a)



(b) Yes. From A, we can reach B in one move and reach C in two moves.

From B, we can reach A and C in one move.

From C, we can reach A and B in one move.

Thus the Markov chain is ergodic.

(c)

$$\text{let } p = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

Then the two step transition matrix is

$$p^2 = \begin{pmatrix} \frac{5}{12} & \frac{5}{12} & \frac{1}{6} \\ \frac{4}{9} & \frac{4}{9} & \frac{1}{9} \\ \frac{5}{12} & \frac{5}{12} & \frac{1}{6} \end{pmatrix} \quad \begin{matrix} \text{1st} \\ \text{2nd} \end{matrix}$$

$$(d) \quad \lambda_0 = (0.3, 0.45, 0.25)^T$$

$$\lambda_{21} = 0.3 \times \frac{5}{12} + 0.45 \times \frac{4}{9} + 0.25 \times \frac{5}{12}$$

$$\lambda_{22} = 0.3 \times \frac{5}{12} + 0.45 \times \frac{4}{9} + 0.25 \times \frac{5}{12}$$

$$\lambda_{23} = 0.3 \times \frac{1}{6} + 0.45 \times \frac{1}{9} + 0.25 \times \frac{1}{6}$$

Thus

$$\lambda_2 = (\lambda_{21}, \lambda_{22}, \lambda_{23}) = (0.42917, 0.42917, 0.14167)$$