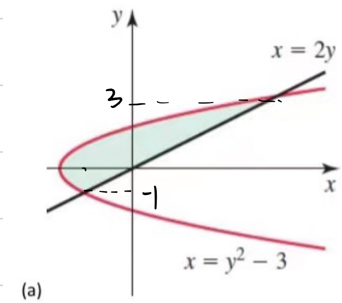
(a)



point of intersection:

$$\begin{cases} x = 2y \\ x = y^2 - 3 \end{cases} \Rightarrow y^2 - 2y - 3 = 0$$

$$(y-3)(y+1) = 0$$

$\therefore y = -1$ 或 $3.$

$$\therefore S = \iint_D dx\,dy = \int_{-1}^{3} \left(\int_{y^2-3}^{2y} dx \right) dy$$

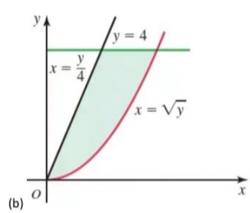
$$= \int_{-1}^{3} 2y - y^2 + 3 \; dy$$

$$= \left(y^2 - \frac{1}{3}y^3 + 3y \right) \Big|_{-1}^{3}$$

$$= (9 - 9 + 9) - (1 + \frac{1}{3} - 3)$$

$$= \frac{32}{3}$$

(b)



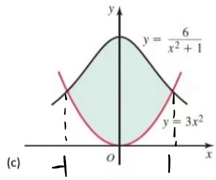
$$S = \int_0^4 \sqrt{y} - \frac{y}{4} \; dy$$

$$= \left(\frac{2}{3} y^{\frac{3}{2}} - \frac{1}{8} y^2 \right) \Big|_0^4$$

$$= \frac{16}{3} - 2$$

$$= \frac{10}{3}$$

(c)



point of intersection:

$$\frac{6}{x^2+1} = 3x^2$$

$$x^4 + x^2 - 2 = 0$$

$$(x^2-1)(x^2+2) = 0$$

$$\therefore x = \pm 1$$

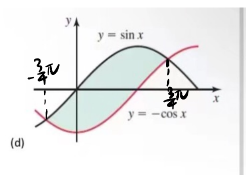
$$S = \int_{-1}^{1} \frac{6}{x^2+1} - 3x^2 \, dx$$

$$= (6\arctan x - x^3)\Big|_{-1}^{1}$$

$$= \left(\frac{6\pi}{4} - 1\right) - \left(-\frac{6\pi}{4} + 1\right)$$

$$= 3\pi - 2$$

(d)



point of intersection:

$$\sin x = -\cos x$$

$$\sin x + \cos x = 0$$

$$\sin\left(x + \frac{\pi}{4}\right) = 0$$

$$x = -\frac{\pi}{4}, \ \frac{3}{4}\pi$$

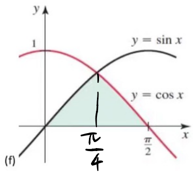
$$S = \int_{-\frac{1}{4}\pi}^{\frac{3}{4}\pi} \sin x - (-\cos x) \, dx$$

$$= (-\cos x + \sin x)\Big|_{-\frac{1}{4}\pi}^{\frac{3}{4}\pi}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$= 2\sqrt{2}$$

(f)



point of intersection:

$$\sin x = \cos x \Rightarrow x = \frac{\pi}{4}.$$

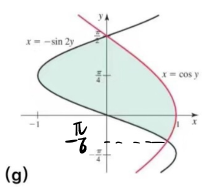
$$\therefore S = \int_0^{\frac{\pi}{4}} \sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x \, dx$$

$$= (-\cos x)\Big|_0^{\frac{\pi}{4}} + (\sin x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left(-\frac{\sqrt{2}}{2}\right) - (-1) + \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$= 2 - \sqrt{2}$$

(g)



point of intersection:

$$-\sin 2y = \cos y.$$

$$(1 + 2\sin y)\cos y = 0.$$

$$\therefore y = \frac{\pi}{2}, -\frac{\pi}{6}$$

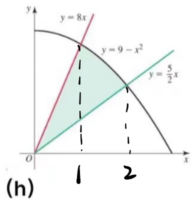
$$S = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos y - (-\sin 2y) \, dy$$

$$= \left(\sin y - \frac{1}{2}\cos 2y\right)\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left(1 + \frac{1}{2}\right) - \left(-\frac{1}{2} - \frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{9}{4}$$

(h)



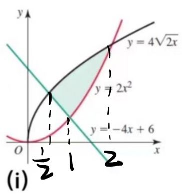
point of intersection:

$$\begin{cases} y=8x \\ y=9-x^2 \end{cases} \Rightarrow x^2+8x-9=0 \Rightarrow x=1$$
$$(x-1)(x+9)=0$$

$$\begin{cases} y=9-x^2 \\ y=\frac{5}{2}x \end{cases} \Rightarrow x^2+\frac{5}{2}x-9=0 \Rightarrow x=\frac{-\frac{5}{2}\pm\sqrt{\frac{25}{4}+36}}{2}=2$$

$$S=\int_0^1\left(8x-\frac{5}{2}x\right)dx+\int_1^2\left(9-x^2-\frac{5}{2}x\right)dx$$
$$=\left(4x^2-\frac{5}{4}x^2\right)\Big|_0^1+\left(9x-\frac{1}{3}x^3-\frac{5}{4}x^2\right)\Big|_1^2$$
$$=\left(4-\frac{5}{4}\right)+\left(18-\frac{8}{3}-5\right)-\left(9-\frac{1}{3}-\frac{5}{4}\right)$$
$$=\frac{47}{3}$$

(i)

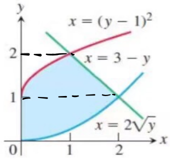


point of intersection:

$$4\sqrt{2x}=-4x+6 \Rightarrow x=\frac{1}{2}$$
$$-4x+6=2x^2 \Rightarrow x=1$$
$$4\sqrt{2x}=2x^2 \Rightarrow x=2$$

$$\therefore S=\int_{\frac{1}{2}}^1\left[4\sqrt{2x}-(-4x+6)\right]dx+\int_1^2\left(4\sqrt{2x}-2x^2\right)dx$$
$$=\left(\frac{4\sqrt{2}\cdot 2}{3}x^{\frac{3}{2}}+2x^2-6x\right)\Big|_{\frac{1}{2}}^1+\left(\frac{8\sqrt{2}}{3}x^{\frac{3}{2}}-\frac{2}{3}x^3\right)\Big|_1^2$$
$$=\left(\frac{8\sqrt{2}}{3}+2-6\right)-\left(\frac{4}{3}+\frac{1}{2}-3\right)+\left(\frac{32}{3}-\frac{16}{3}\right)-\left(\frac{8\sqrt{2}}{3}-\frac{2}{3}\right)$$
$$=\frac{19}{6}$$

(j)



(j)

point of intersection:

$(y-1)^2 = 3-y \Rightarrow y=2$

$3-y = 2\sqrt{y} \Rightarrow y=1.$

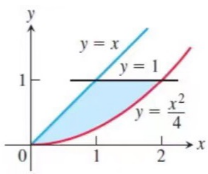
$$S=\int_0^1 2\sqrt{y}\,dy + \int_1^2 [3-y-(y-1)^2]dy$$

$$= \left(\frac{4}{3}y^{\frac{3}{2}}\right)\Big|_0^1 + \left[3y-\frac{1}{2}y^2-\frac{1}{3}(y-1)^3\right]\Big|_1^2$$

$$= \frac{4}{3} + \left(6-2-\frac{1}{3}\right)-\left(3-\frac{1}{2}\right)$$

$$= \frac{17}{2}$$

(k)



(k)

$y=\frac{x^2}{4} \Rightarrow x=\sqrt{4y}=2\sqrt{y}$

$$S=\int_0^1 2\sqrt{y}-y\,dy$$

$$=\left(\frac{4}{3}y^{\frac{3}{2}}-\frac{1}{2}y^2\right)\Big|_0^1$$

$$=\frac{4}{3}-\frac{1}{2}$$

$$=\frac{5}{6}$$